Dynamical Evolution of Rotating Stellar Systems: Fokker-Planck and N-body

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A globular cluster M15

Stellar Systems

- Star clusters (open, globular), galactic nuclei, galaxies, clusters of galaxies
- Self-gravitating
- Composed of mostly point masses
- Weak field (i.e., Newtonian dynamics)
- Question: What is the fate of the self-gravitating system of point masses interacting according to Newton's laws?

Globular Clusters Dynamics

- Oldest objects in the Galaxy: astronomical fossil
- Mostly “spherical”: ideal target for study of N-body dynamics
- Statement of problem:

What is the fate of the self-gravitating system of point masses interacting according to Newton's laws?

Thermodynamic properties

$$\text{Moment of inertia}: I = \eta M r^2$$

$$\text{Temperature}: kT = \frac{1}{2} m \sigma^2$$

virial theorem:

$$E = K + V = -3NkT = -\zeta \frac{GM^2}{r}$$

$$\text{specific heat}: \frac{\partial E}{\partial T} < 0$$

$$\text{specific moment of inertia}: \frac{\partial I}{\partial T} < 0$$

self-gravitating systems are thermodynamically unstable

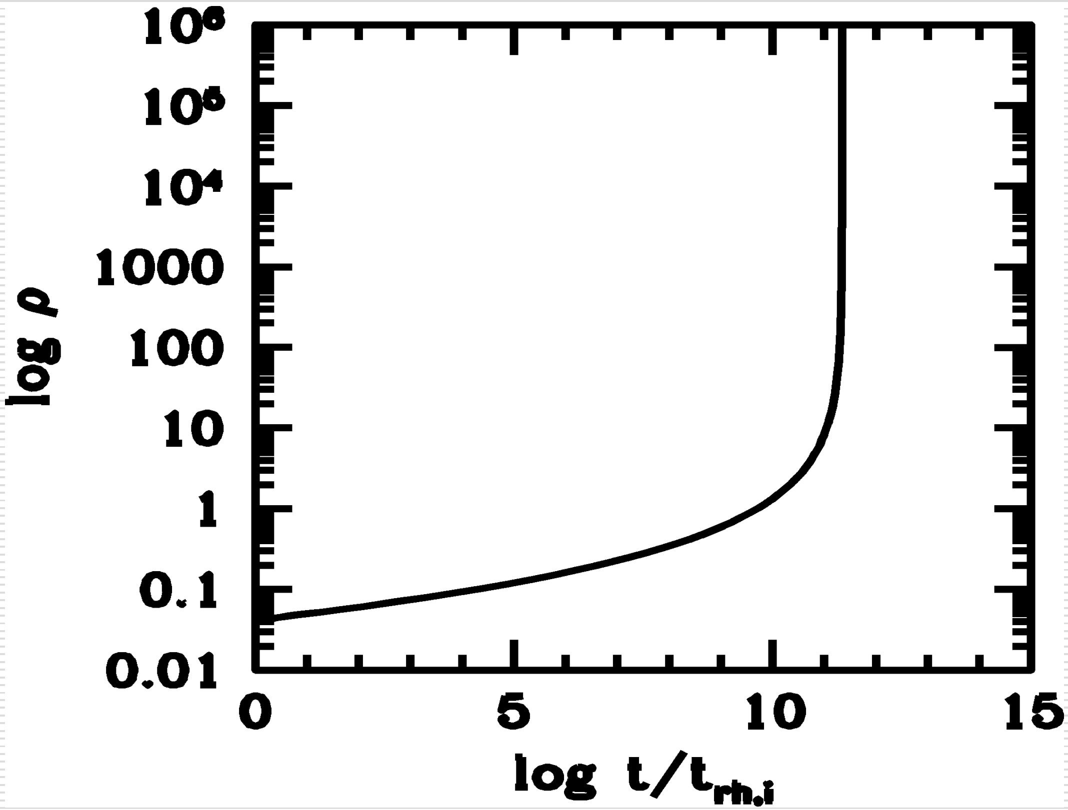
Consequences

- The negative specific heat drives the "core collapse"
- The negative specific moment of inertia make the system unstable, and thus further drives the collapse of the central part.
- More detailed course of evolution can be followed only by numerical methods

General Evolution of Simple, Self-gravitating Systems

- The central density becomes infinite in finite time: gravothermal catastrophe.
- In real systems, infinite density would never happen
- Many collisional effects become important

Based on Fokker-Planck equation



Methods of Dynamical Evolution

- Direct integration of equation of motion (N-body method): very time consuming ,

  only up to $N<10^5$ systems, whereas we are interested in $N> 10^6$ systems

- Statistical methods: Fokker-Planck equation, Gas dynamical approach, Monte Carlo method: Fast, but many restrictions

  spherical symmetry

  velocity dispersion tensor

  external field

Integrals of Motion and Jeans Theorem

Integrals of motion: a set of constants of motion that isolates the orbital properties :

- Spherical system: Energy (E) and angular momentum (J)

- Spherical system with isotropic velocity: E

- Rotating Axisymmetric systems: E, J along the axis of rotation (J_z), and I_3 (unknown third integral).

Jeans Theorem: The distribution function of collisionless systems depends on phase space variables only through the integrals of motion

Adiabatic Invariants

- Physical quantities that remain nearly constant for slowly varying potential.
- In spherical potential, radial action Q(E,J), in addition to J, is an adiabatic invariant.

$$Q = 2\int_{r_-}^{r_+} v_r \, dr$$

$f(Q, J)$ remains fixed with small changes in potential

Fokker-Planck Equation 1.

- Jeans Theorem:

$$f = f(I_1, I_2, \ldots; \quad t)$$

- Effects of Stellar Encounters

The collisional effects are assumed to be dominated by distant encounters (accumulation of small changes in velocity).

Evolution of distribution function in time

→ Diffusion equation in phase space

Fokker-Planck equation 2.

Schematic form of Fokker-Planck equation in a flux conserved form

$$4\pi^2 p \frac{\partial f}{\partial t} = -\frac{\partial}{\partial I_1} F_{I_1} - \frac{\partial}{\partial I_2} F_{I_2} - ....$$

where F_I is the particle flux in integral I.

$$F_{I_i} = -D_{I\,i} f - D_{I_i I_j} \frac{\partial f}{\partial I_j}$$

D_I's are called the Fokker-Planck coefficients

Fokker-Planck equation in self-gravitating systems

- As the system evolves, the gravitational potential also changes

→ integrals (I) may change

- The Fokker-Planck equation assumes that the potential is fixed: we need some modification of the simple Fokker-Planck equation

- Self-consistent distribution function and gravitational potential must be computed

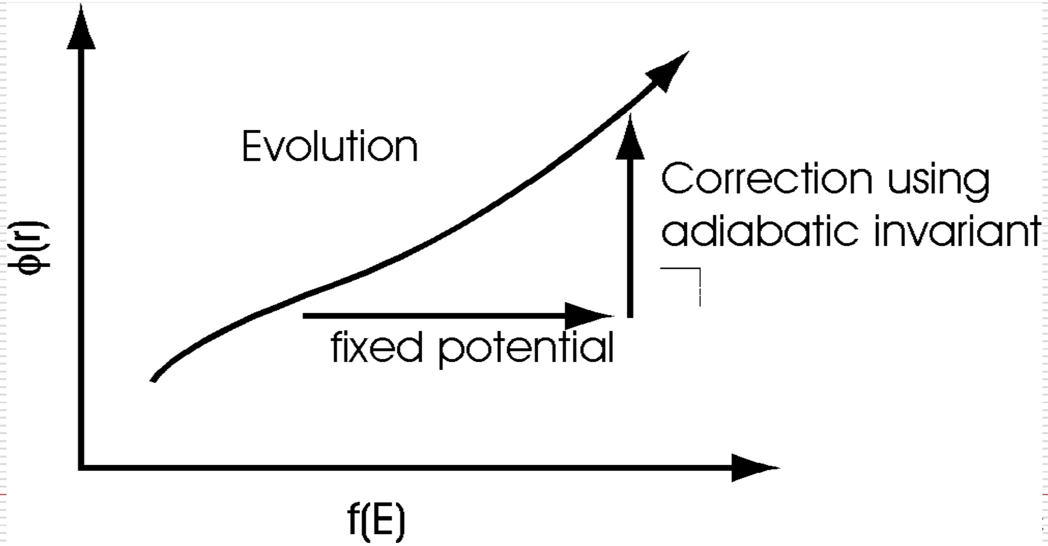
→ utilize the adiabatic invariants

A Procedure to obtain self-consistent potential

$$f^{new}(\phi_{old}) \rightarrow \rho(f) \rightarrow \phi$$

$$\uparrow$$

$$\xrightarrow{f(Q,J)} f(E,J)$$



Application of Fokker-Planck equation

- Fokker-Planck equation has been widely used for the study of spherical systems including the effects of

  - Stellar Evolution
  - Formation/destruction of binaries
  - Inelastic (superelastic) encounters
  - External Field (constant or time varying)

→ Modification of flux functions and addition of source and sink terms

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial E}\left(-D_E f - D_{EE}\frac{\partial f}{\partial E} - H_E f - H_{EE}\frac{\partial f}{\partial E} \right)$$

$$-Lf + A$$

Rotating Systems

Why do we care?

- Rotation is ubiquitous among astronomical objects (tidal effects…)
- Evidences for rotation in star clusters
- What is the consequence of the rotation?

(i.e., role of gravo-gyro instability)

Difficulties

- Three isolating integrals
- Not spherically symmetric anymore

Fokker-Planck equation for rotating systems

- Integrals: E, J_z
- Third integral is unknown and ignored

$$4\pi^2 p\frac{\partial f}{\partial t} = -\frac{\partial}{\partial I_1}F_{I_1} - \frac{\partial}{\partial I_2}F_{I_2} - \ldots$$

Interesting Issues

- Effects of rotation on dynamical evolution
- Angular momentum redistribution/ transport
- Comparison with observations
- Does Fokker-Planck equation work well for rotating systems?

Rotating Fokker-Planck Code

- The basic code was developed by Einsel & Spurzem (1999)

(Since the third integral is unknown, it is ignored)

- The Poisson equation is solved in cylindrical coordinate with azimuthal symmetry.
- Substantially improved and collisional effects, tidal boundary conditions, etc have been added by E. Kim, H. M. Lee etc.

Initial Configuration

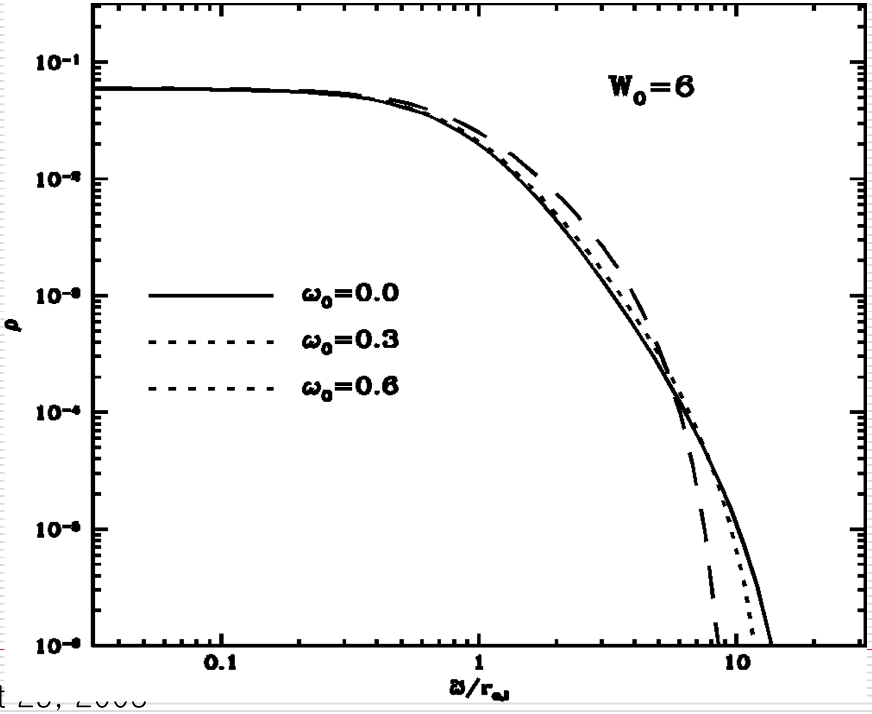
Rotating King Models:

$$f(E,J_z)=a\left(e^{-\beta E}-1\right)\times e^{-\beta\Omega_0 J_z}$$

Two parameter family of models

Central potential: $W_0=\phi_0/\sigma^2$

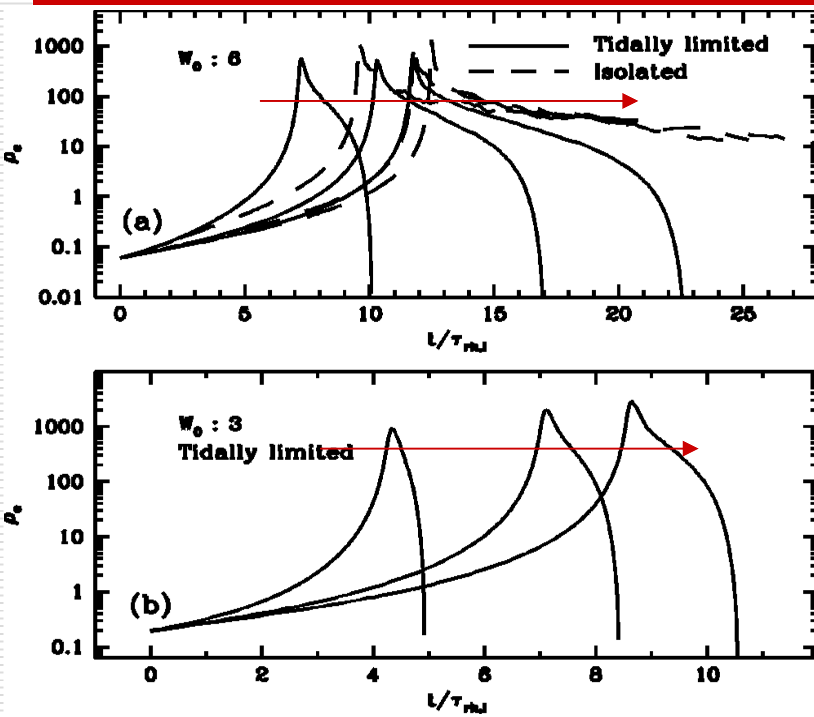
Rotational parameter: $\omega_0=\dfrac{3\Omega_0}{\sqrt{4\pi G n_c}}$



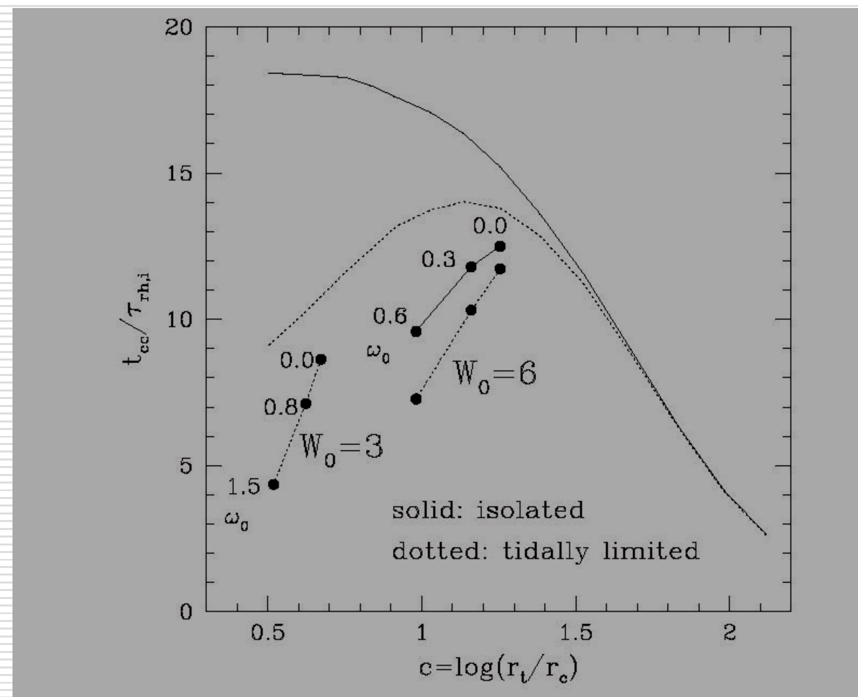
Properties of Initial Models

W_0	ω_0	r_h/r_t	$T/\|W\|$	e_{dyn}
6	0.0	0.15	0.0	0.0
	0.18	0.035	0.101	0.285
	0.6	0.24	0.101	0.285
3	0.0	0.26	0.0	0.0
	0.8	0.29	0.035	0.102
	1.5	0.35	0.097	0267

Evolution of rotating clusters



Direction toward higher rotation



Rotation makes the evolution faster

Kim et al. 2002

Evaporation of Stars

The stars evaporate from cluster when
$v > v_e$, where $v_e = -2\Phi$ is escape velocity.

The mean square of escape velocity

$$< v_e^{\ 2} > = \frac{\int \rho(x) v_e^{\ 2} d^3x}{\int \rho(x) d^3x} = -2 \frac{\int \rho(x)\Phi(x) d^3x}{M}$$
$$= \frac{-4W}{M}$$

where W is potential energy, and M is the total mass.

According to Virial Theorem,

$$-W = 2K = \frac{1}{2} M < v^2 >,$$

and therefore,

$$< v_e^{\ 2} > = 4 < v^2 >.$$

Evaporation Rate

Suppose the stellar system reaches velocity distribution $f(v)$ in relaxation time (t_{rh}).

The evaporation rate per relaxation time becomes,

$$\xi_e = -\frac{t_{rh}}{M}\frac{dM}{dt} = \frac{\int_{v_e}^{\infty} f(v)d^3v}{\int_0^{\infty} f(v)d^3v}.$$

If $f(v)$ is a Maxwellian, $\xi = 0.00738$

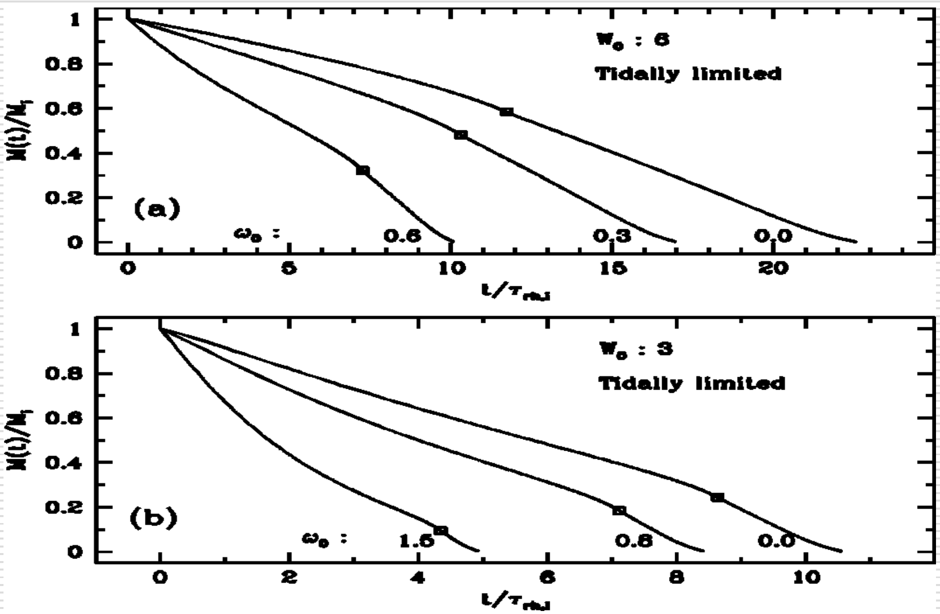
For tidally bounded systems, the escape velocity is reduced:

$$<v_e^2> = 4(1-\lambda) < v^2 >$$

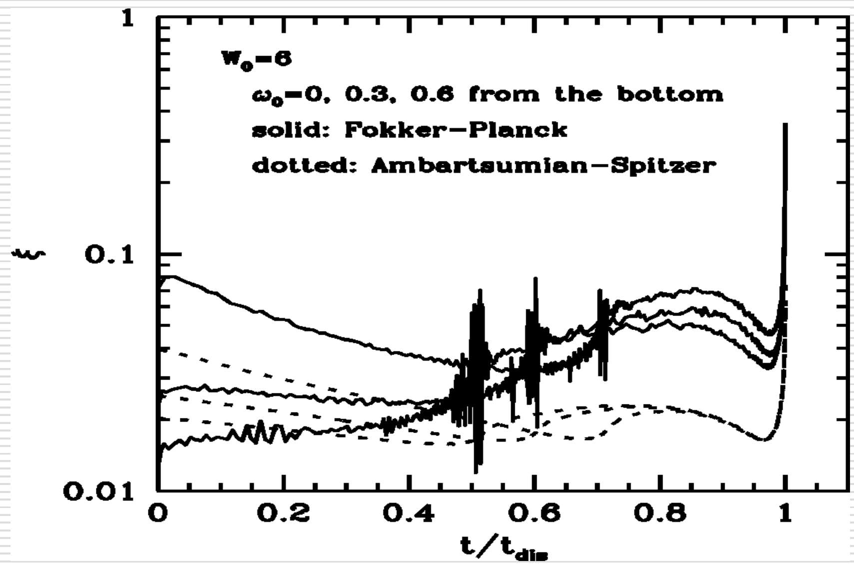
where

$$\lambda = \frac{GM}{r_t} \Big/ \frac{0.8GM}{r_h} = \frac{5r_h}{4r_t}$$

Evaporation is also accelerated

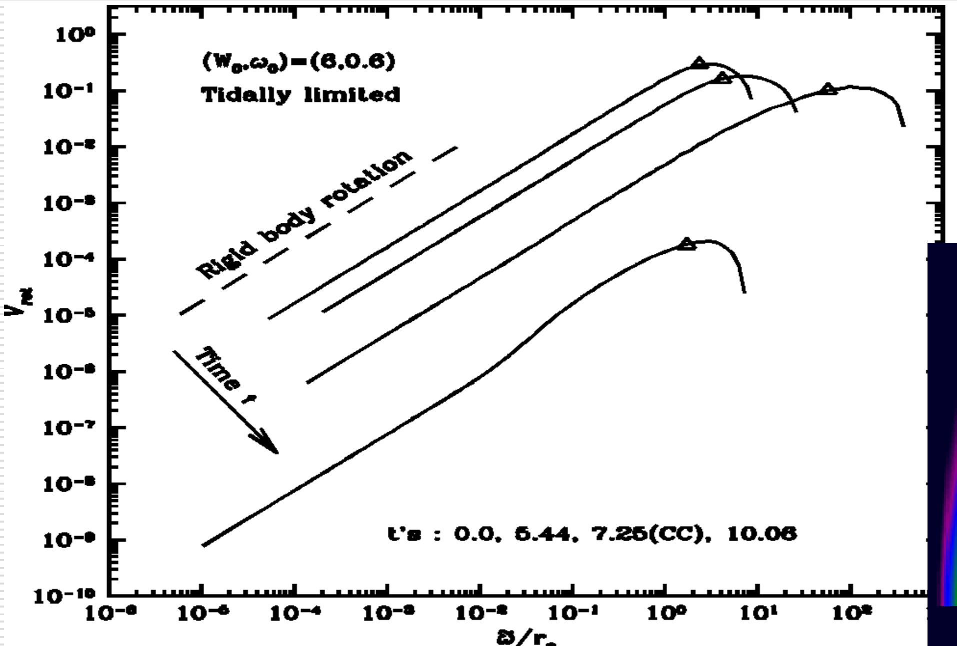


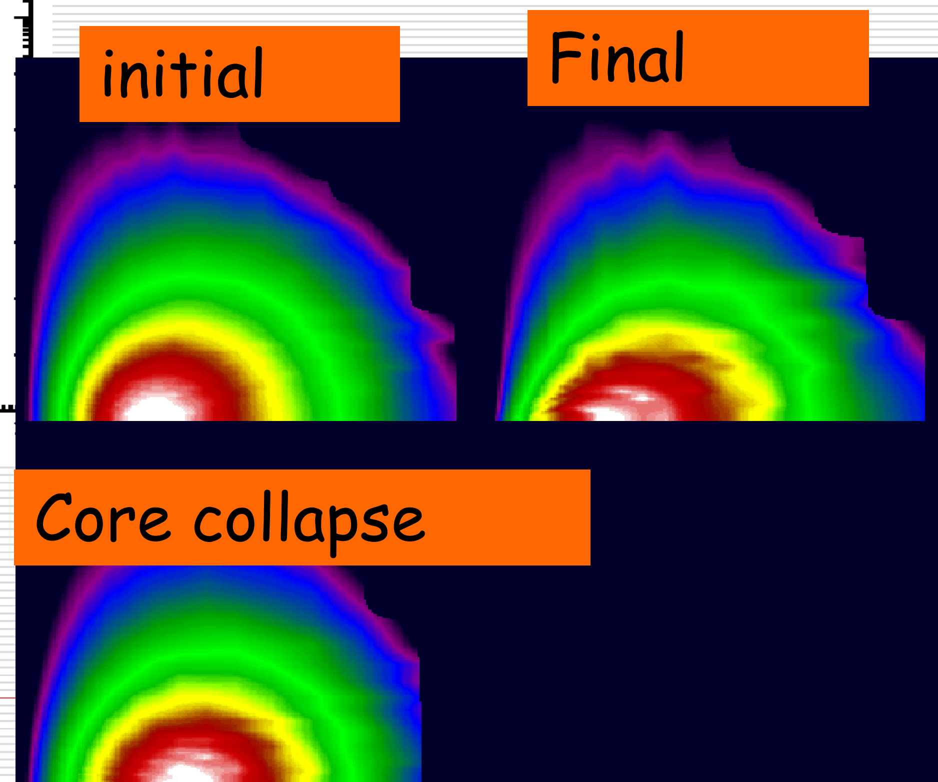
Initial evaporation is greatly enhanced by rotation



Kim et al. 2002

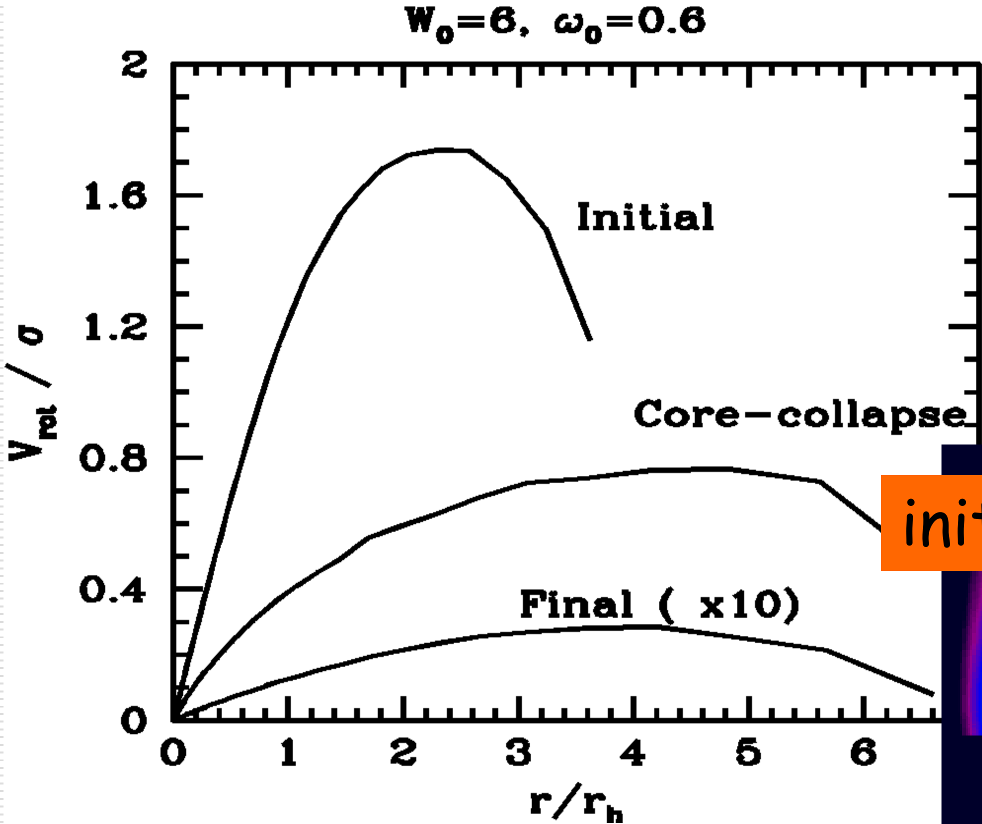
Evolution of Rotation 1.

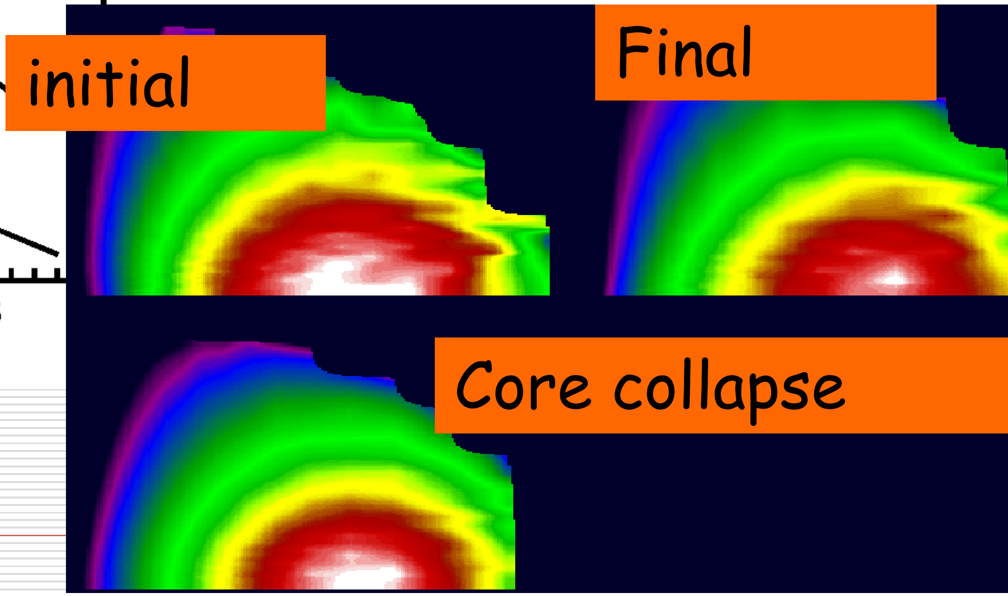




Shape of rotation curve does not change, but the rotation velocity decreases

Evolution of Rotation 2.

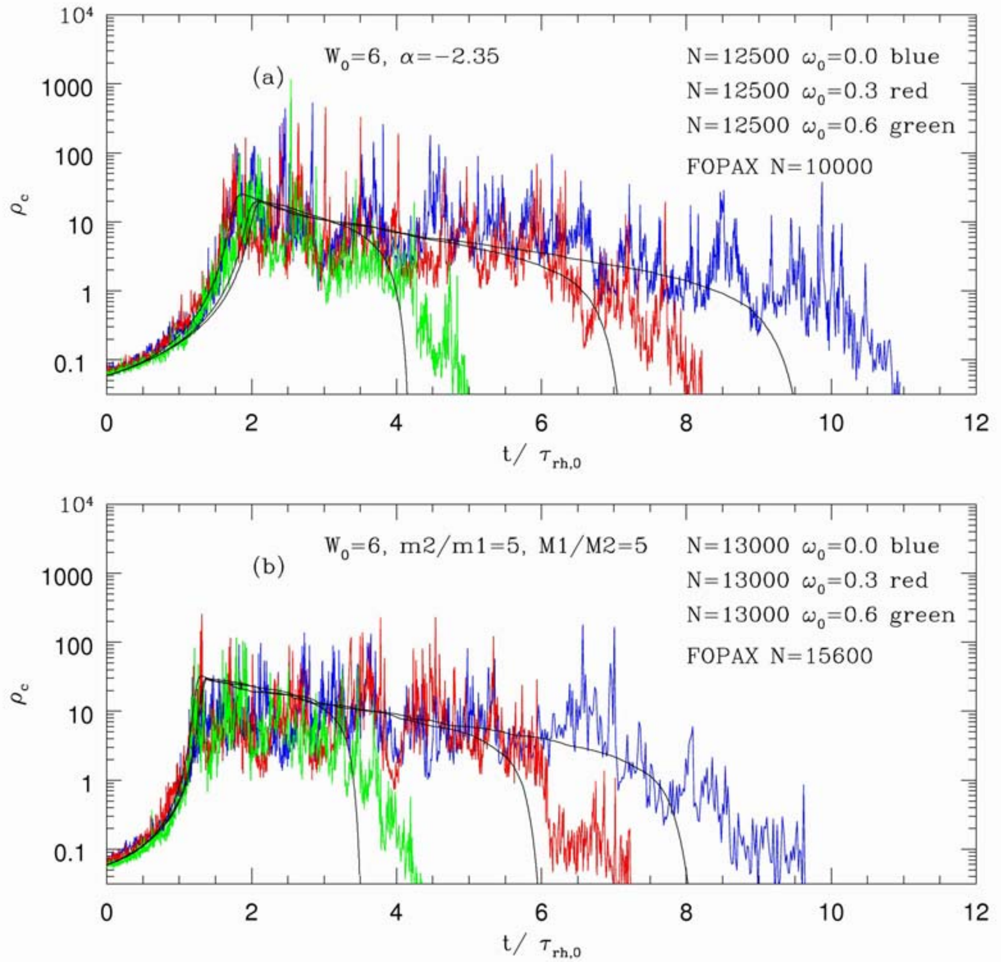




Comparisons with N-body

- Fokker-Planck equation ignores the third integral: could it be justified?
- Comparisons with N-body is necessary to assure the applicability of the Fokker-Planck approach
- We have carried out N-body simulations of initially rotating clusters with same initial conditions, for single and multi-mass cases

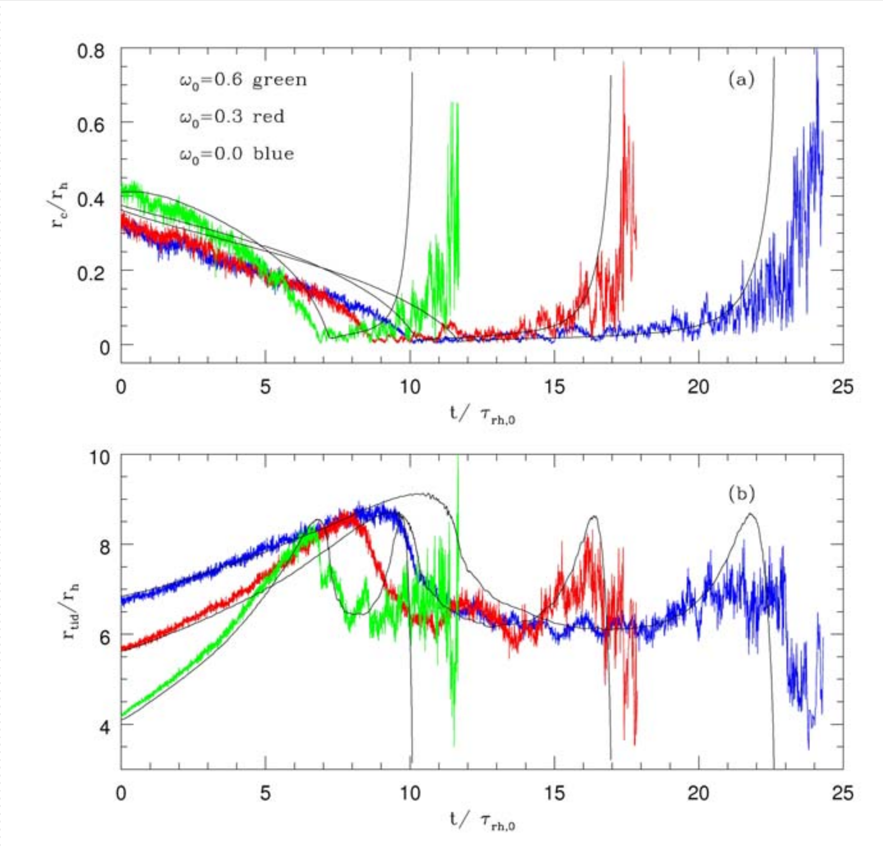
Evolution of central density

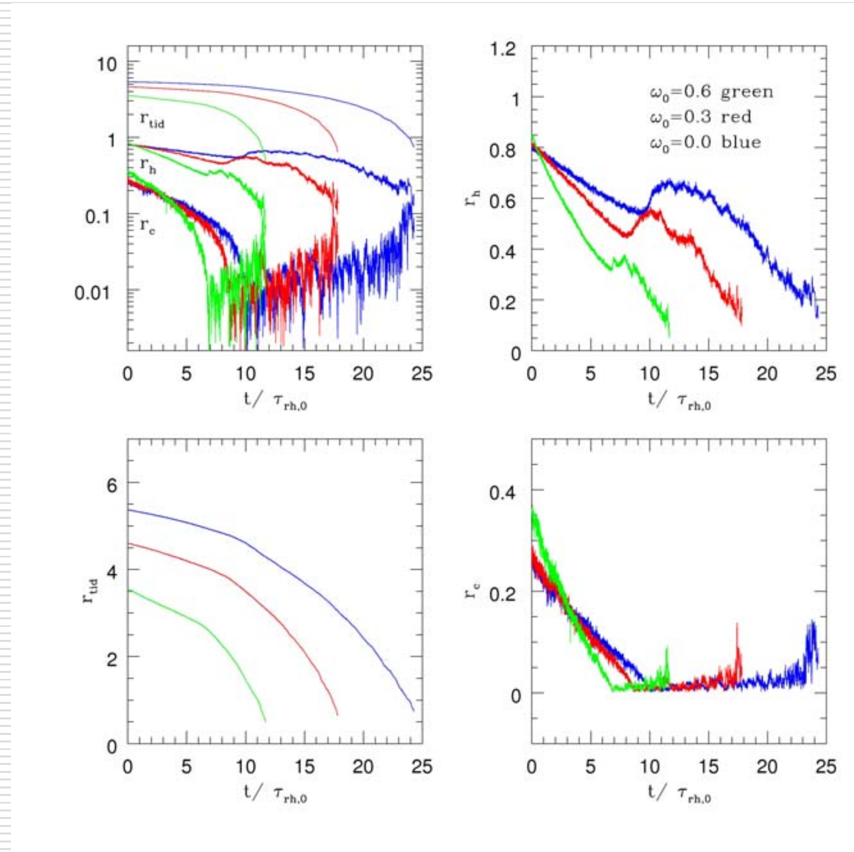


N-body and FP results are similar. FP models are disrupted more quickly: tidal boundary condition

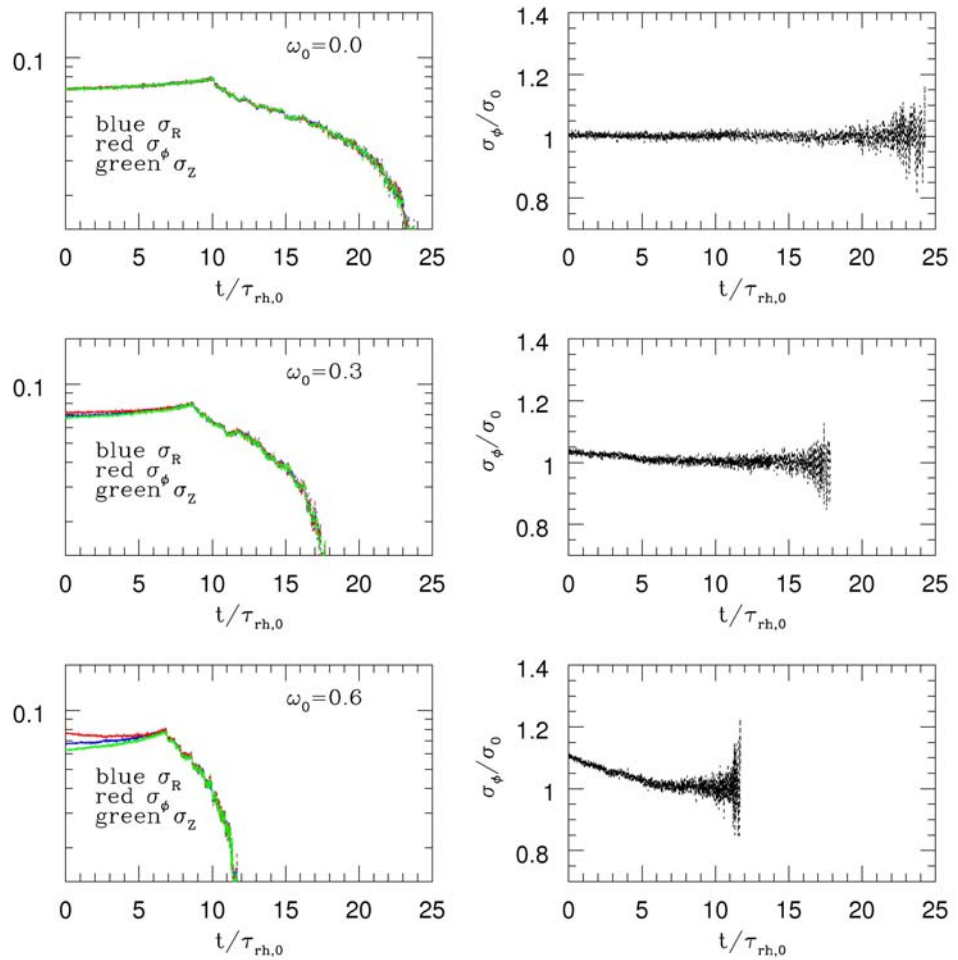
Kim et al. 2006

Evolution of Characteristic Radii





Velocity Profiles

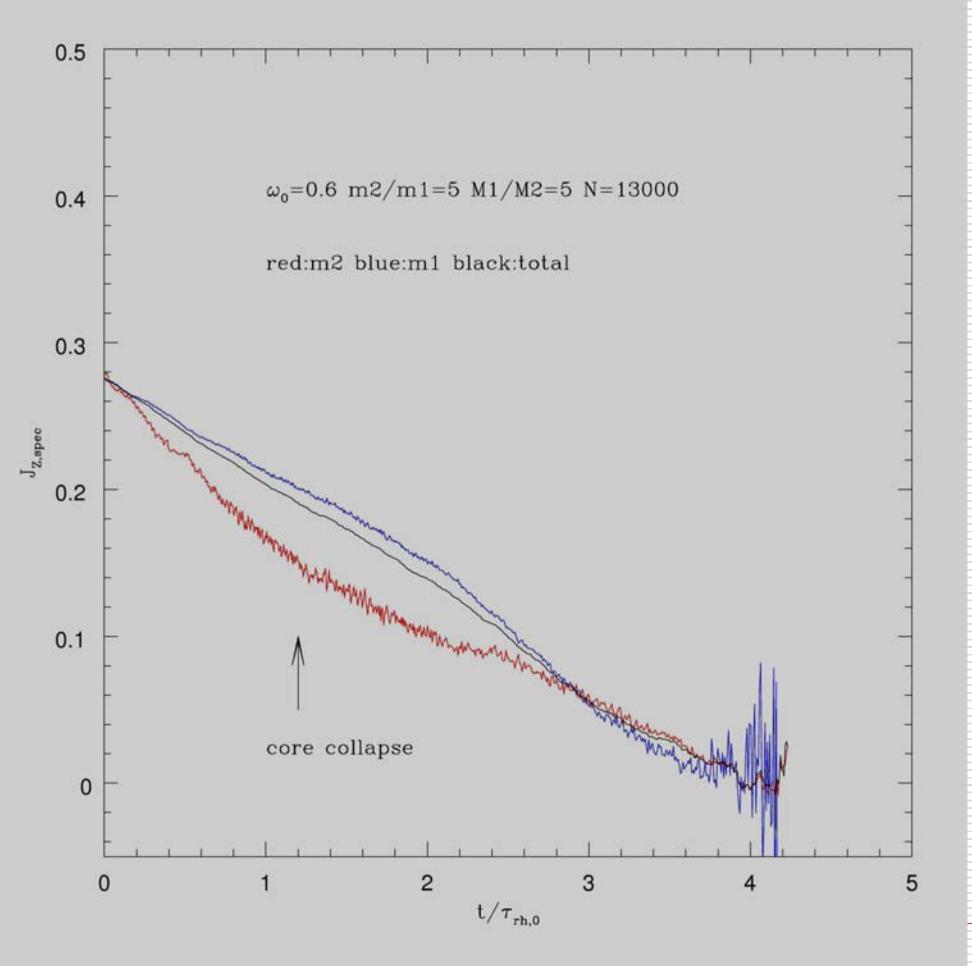


Velocity distribution becomes isotropic due to two-body relaxation: the third integral would eventually disappear

Multi-Mass Models

- Multi-mass models are more complex: the energy exchange among different mass components could accelerate the evolution
- FP calculations for multi-mass rotating clusters are carried out by Kim et al. 2004
- N-body calculations by Kim et al. 2006 confirmed most of the features of FP calculations

Example: Evolution of specific angular momentum



Angular momentum for higher mass component decreases more rapidly in the early phase

Summary and Implications

- Direct integration of Fokker-Planck equations, ignoring the third integral have been extensively carried out with tidal cut-off, binary heating, and mass spectrum
- Rotation tends to accelerate the evolution of star clusters due to ‘gravo-gyro’ instability
- There exists a competition in the acceleration of the evolution between the rotation and energy exchange for multi-mass models
- Direct N-body integrations for N~10,000 give very close results with FP: ignorance of the third integral may be justified
- More realistic calculations are carried out/planned (i.e., central black hole, etc,)