General Relativistic Simulation of Jet Formation in Kerr Black Hole Magnetosphere

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(1) Ideal general relativistic MHD (ideal GRMHD) simulation:
Jet formation by magnetic bridges between the ergosphere and disk around a rapidly black hole:
Anti-parallel magnetic field ⇒ Magnetic reconnection

(2) GRMHD with finite conductivity (σGRMHD):
Numerical method and simple tests

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Relativistic Jets in the Universe

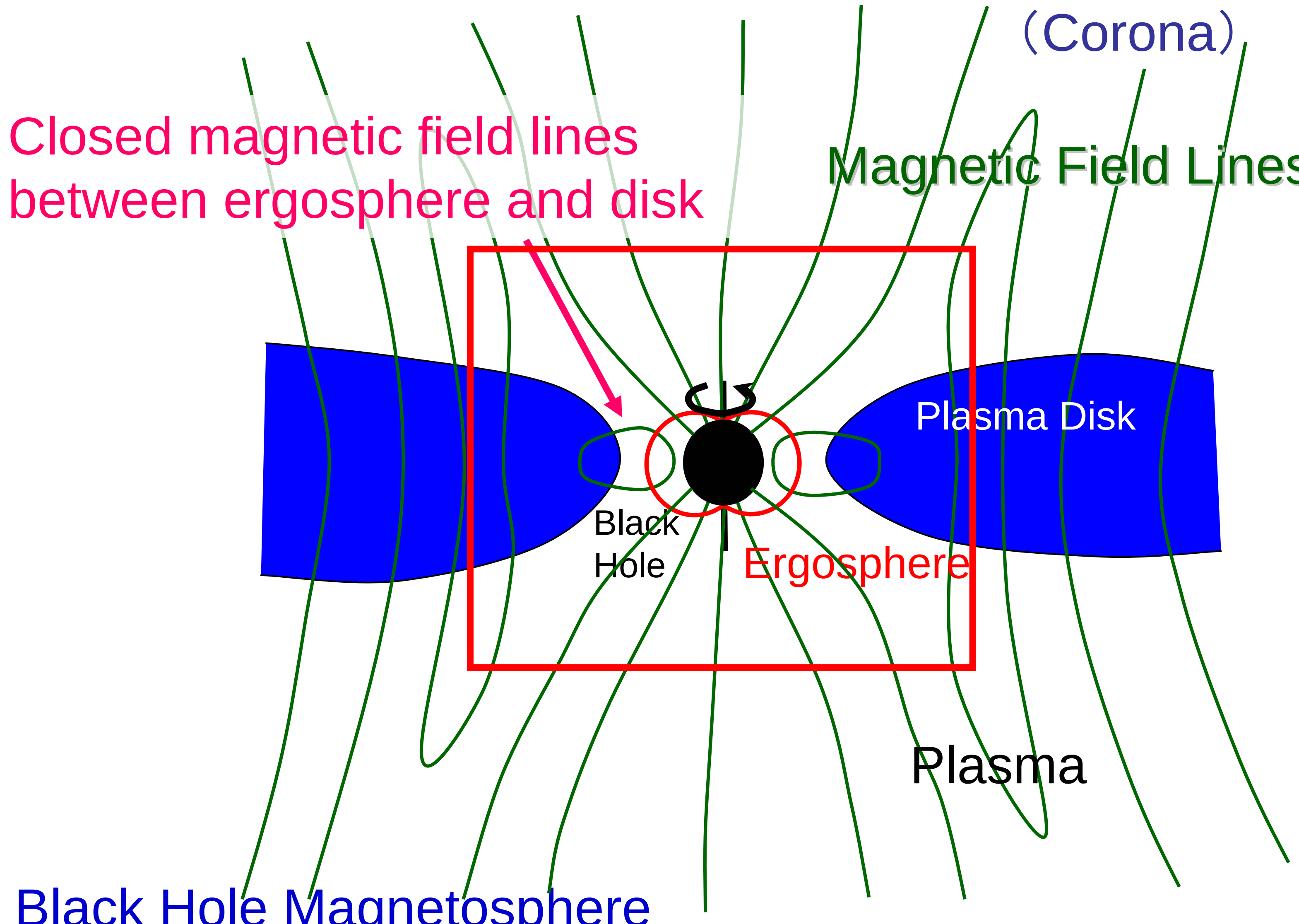
- Active galactic nuclei, Quasars:
  $\gamma \gtrsim 10$, L_{jet}~ several M pc
- Stellar mass black hole binaries (Microquasars):
  $\gamma \sim 3$, L_{jet} ~ several pc
- Gamma-ray bursts: $\gamma \gtrsim 100$, L_{jet} ~ 1AU-several pc

The jet formation mechanism may be common. These relativistic jets are formed by drastic phenomena around black holes. However, distinct model has not yet shown.

Points of models { Acceleration of plasma/gas; Collimation of plasma/gas outflow }

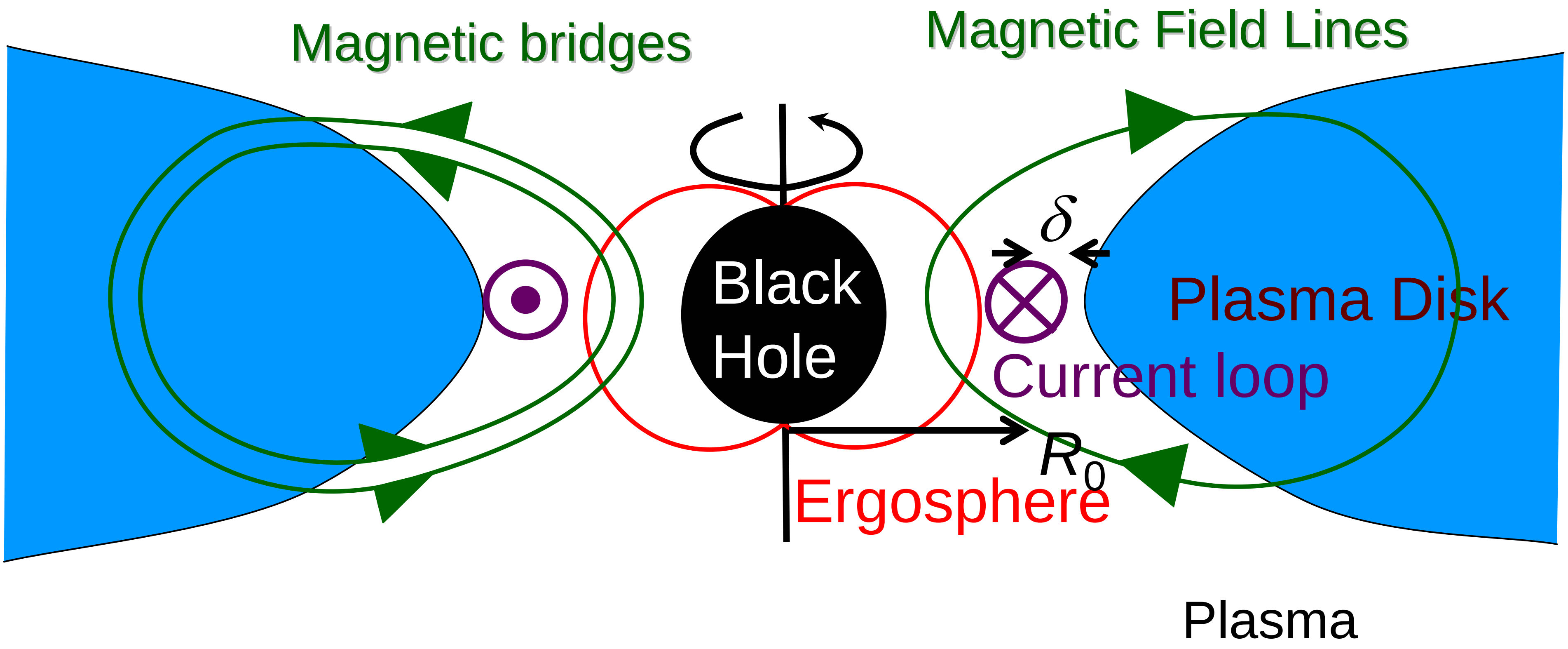
1) Magnetic field
2) Radiation pressure
3) Gas pressure

Black Hole Magnetosphere



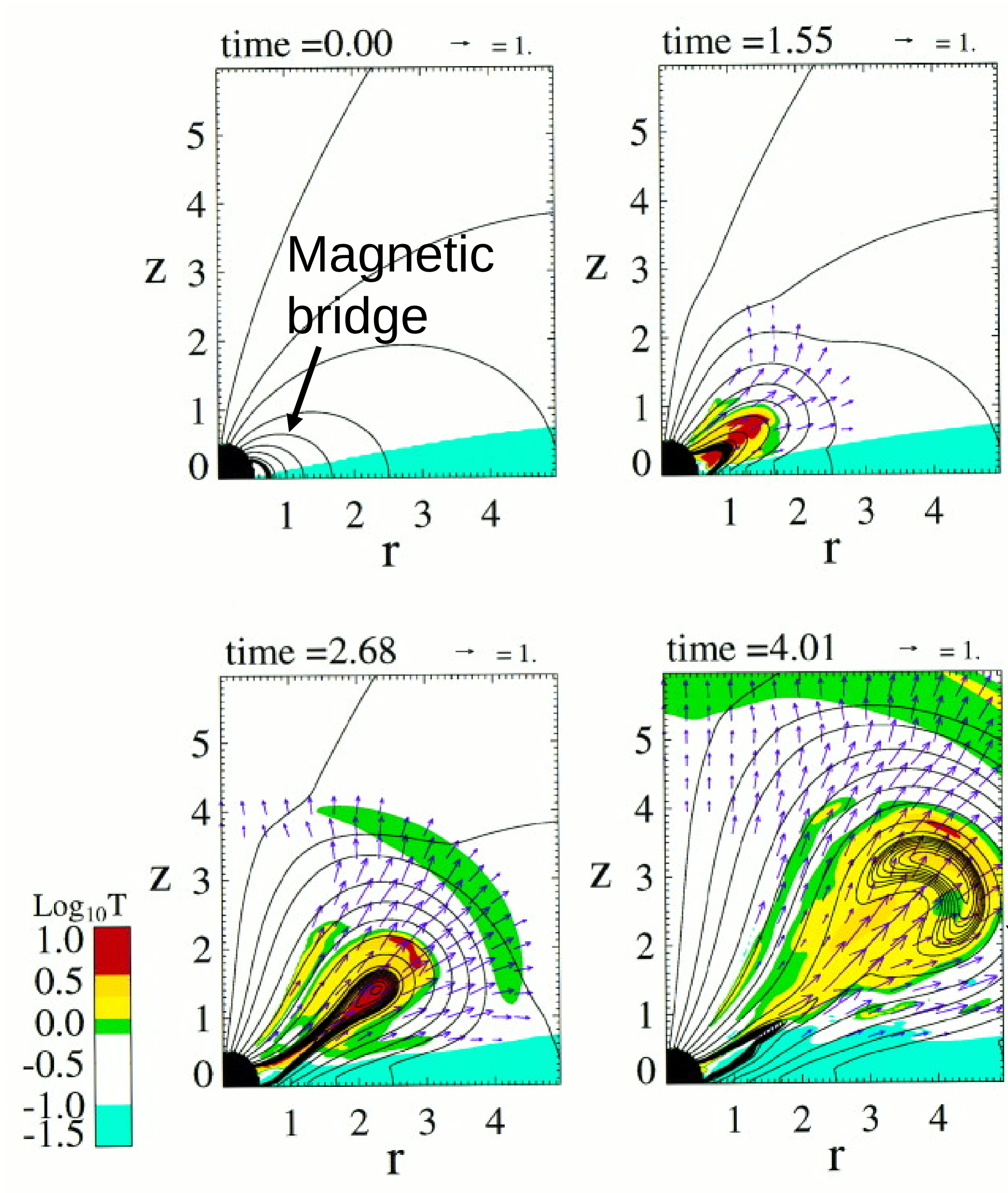
Black Hole Magnetosphere

Magnetic Field induced by Current Loop around Black Hole



Nonrelativistic MHD Simulation with Dipole-Magnetic Field and Disk

Hayashi, Shibata, and Matsumoto (1996)

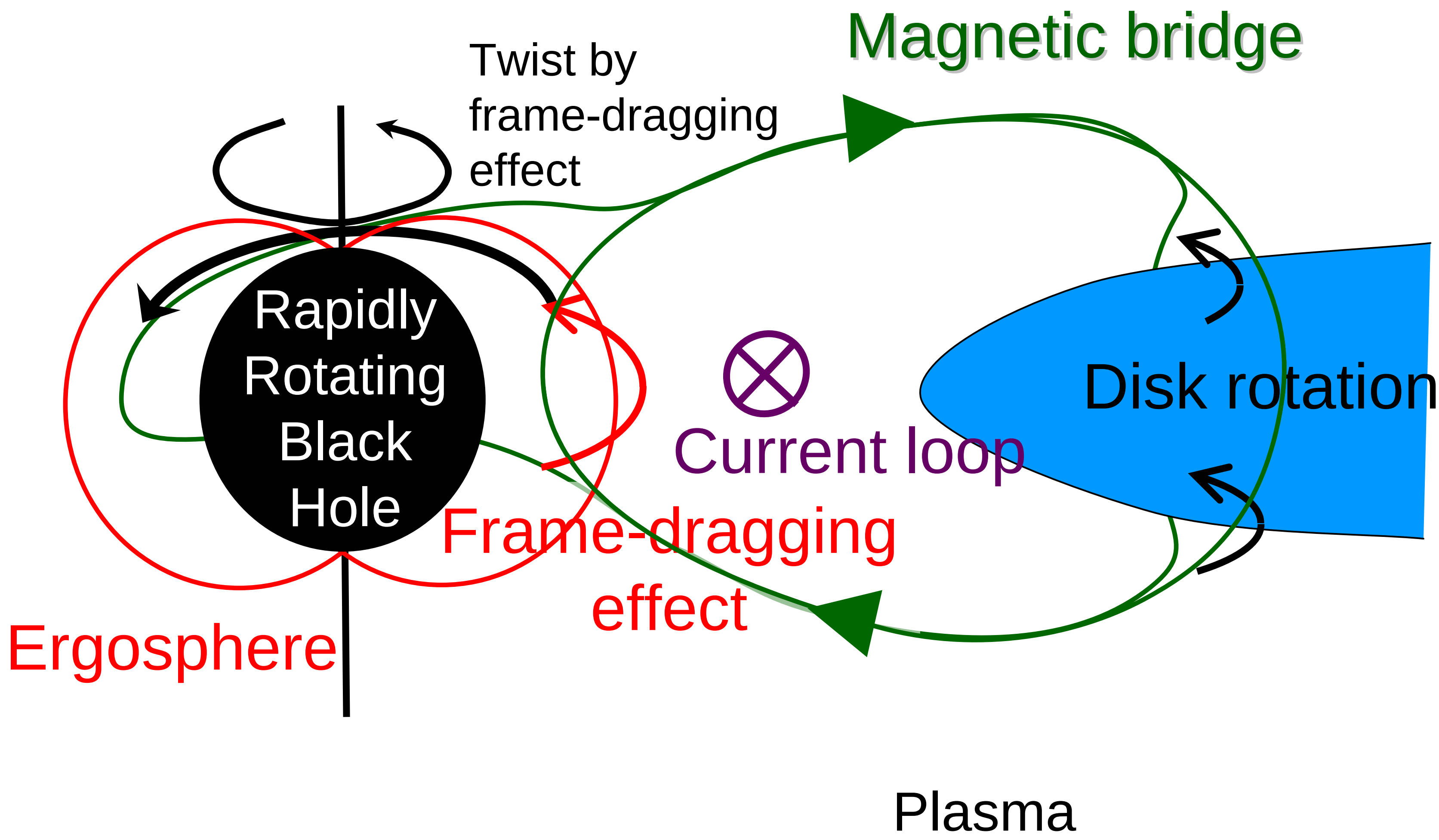


Anomalous resistivity:

$$\sigma = 100 \quad (J/\rho > v_d)$$
$$\sigma = \infty \quad (J/\rho \le v_d)$$

Magnetic island (Plasmoid)

Twist of magnetic bridge by ergosphere



Ideal General Relativistic Magnetohydrodynamics

(Ideal GRMHD)

To investigate dynamics of the magnetic bridge between the ergosphere and the disk, we have to consider the interaction of the plasma and magnetic field near the black hole. Simplest approximation for it is given by ideal GRMHD where electric conductivity σ is infinite ($\sigma \rightarrow \infty$).

Ideal GRMHD Equations in Kerr Space-Time

General relativistic equations of conservation laws:

$$\nabla_\nu \left(nU^\nu\right) = 0$$ (particle number)

$$\nabla_\nu T^{\mu\nu} = 0$$ (energy and momentum)

Maxwell equations:

$$\partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} + \partial_\lambda F_{\mu\nu} = 0$$

$$\nabla_\nu F^{\mu\nu} = -J^\mu$$

Ohm's law with infinite conductivity: $F_{\mu\nu} U^\nu = 0$

Kerr Metric: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

$$g_{00} = -h_0^2; \quad g_{ii} = h_i^2; \; g_{0i} = -h_i^2 \omega_i \; (i = 1,2,3); \quad g_{ij} = 0 \; (i \neq j)$$

n: proper particle number density
U^μ : velocity four vector.
J^μ : current density four vector
∇^μ : covariant derivative.
$T^{\mu\nu}$: energy momentum tensor
$F_{\mu\nu}$: field-strength tensor

3+1 Formalism of Ideal GRMHD Equation
～ similar to nonrelativistic ideal MHD

(conservative form)

Special relativistic mass density, γρ

$$\frac{\partial D}{\partial t} = -\nabla \cdot [\alpha D(\mathbf{v} + c\boldsymbol{\beta})]$$ (conservation of particle number)

general relativistic effect

Special relativistic total momentum density

$$\frac{\partial \mathbf{P}}{\partial t} = -\nabla \cdot [\alpha(\mathbf{T} + c\beta\mathbf{P})] - \left(D + \frac{\varepsilon}{c^2}\right)\nabla(c^2\alpha) + \alpha\mathbf{f}_{\mathrm{curv}} - \mathbf{P} : \boldsymbol{\sigma}$$ (equation of motion)

special relativistic effect

Special relativistic total energy density

$$\frac{\partial \varepsilon}{\partial t} = -\nabla \cdot [\alpha(c^2\mathbf{P} - Dc^2\mathbf{v} + \varepsilon c\boldsymbol{\beta})] - (\nabla\alpha)\cdot c^2\mathbf{P} - \mathbf{T} : \boldsymbol{\sigma}$$ (equation of energy)

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times [\alpha(\mathbf{E} - c\boldsymbol{\beta} \times \mathbf{B})]$$

$$\mathbf{J} + \rho_{\mathrm{e}} c\boldsymbol{\beta} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \left[\alpha\left(\mathbf{B} + \frac{\boldsymbol{\beta}}{c} \times \mathbf{E}\right)\right]$$

No coupling with other Eqs

$$\nabla \cdot \mathbf{B} = 0 \qquad \rho_{\mathrm{e}} = \frac{\alpha}{c^2}\nabla \cdot \mathbf{E}$$ (Maxwell equations)

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$ (ideal MHD condition)

where $\alpha = \sqrt{h_0^2 + \sum_{i=1}^{3}\left(\frac{h_i\omega_i}{c}\right)^2}$: (Lapse function), $\beta^i = \frac{h_i\omega_i}{c\,\alpha}$: (shift vector)

Numerical Method

- The equations of ideal GRMHD are similar to those of nonrelativistic ideal MHD. Therefore, we can use the numerical techniques developed for nonrelativistic ideal MHD calculations. In this study, we use simplified TVD method.
- Simplified TVD method
    - This method is developed by Davis (1984) for hydrodynamic shock wave simulations.
    - Merit: We don't need eigen-vector of Jacobian matrix of equations like primary TVD scheme. Just maximum of eigen-value of the Jacobian is used. It is easily applied for complex equations like GRMHD equations.

Initial Condition

- Black Hole: $a \equiv \frac{J}{J_{\max}} = 0.99995$ (Almost maximally rotating)
- Magnetic Field: Magnetic field induced by current loop around black hole $\left(J_0 = \frac{1.5\pi}{2}, R_0 = r_S, \delta = 0.5 r_S \right)$

  Current — Major radius of current loop — Minor radius

- Plasma:
  - Corona

    hydrostatic equilibrium $(\Gamma_0 = 4, \rho_0 = 0.018)$

    $$\rho_{\rm cor} = \rho_{\rm eq} = \rho_0 \left[\alpha^{-\Gamma/(\Gamma_0+1)(\Gamma-1)} - 1 \right]^{\Gamma_0}$$

    $$p_{\rm cor} = p_{\rm eq} + p_{\rm bg} = \frac{\Gamma - 1}{\Gamma} \rho_0 c^2 \left[\alpha^{-\Gamma/(\Gamma_0+1)(\Gamma-1)} - 1 \right]^{\Gamma_0+1} + \frac{p_{\rm bg}^0}{\alpha}$$

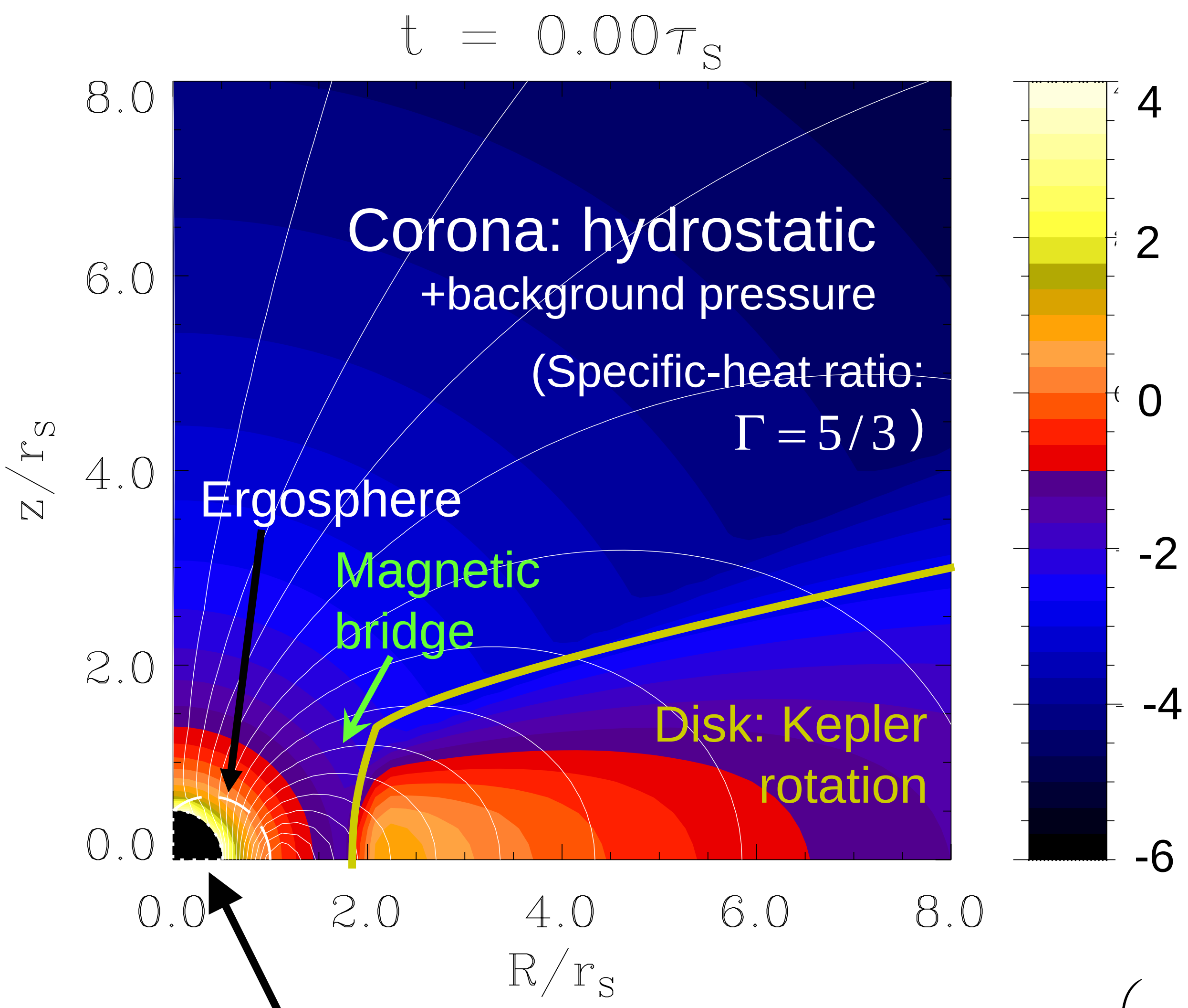
    $$\hat{\mathbf{v}} = 0$$

  - Disk (Co-rotating disk)

    $\rho_{\rm disk} = 1{,}000\, \rho_{\rm cor}$, $p_{\rm disk} = p_{\rm cor}$

    $\hat{\mathbf{v}}_{\rm P} = 0$, $v_\phi = v_{\rm Kepler}$

Initial condition of Ideal GRMHD simulation



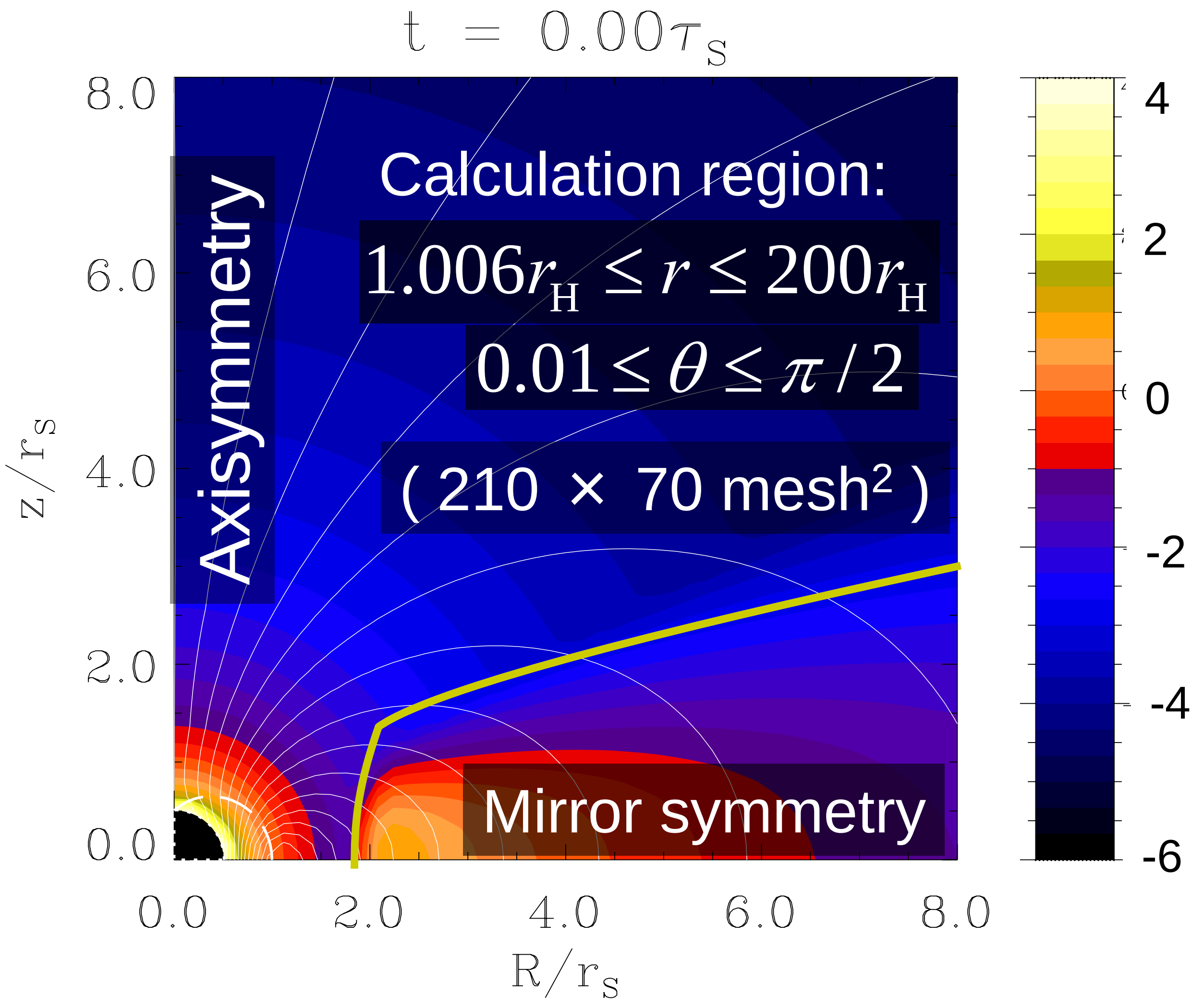
t =0

Solid white line:
Magnetic field line

Color: $\log \rho$

Almost maximally rotating Black hole $\left(a \equiv \frac{J}{J_{\max}} = 0.99995 \right)$

Condition of Ideal GRMHD simulation



$t = 0$

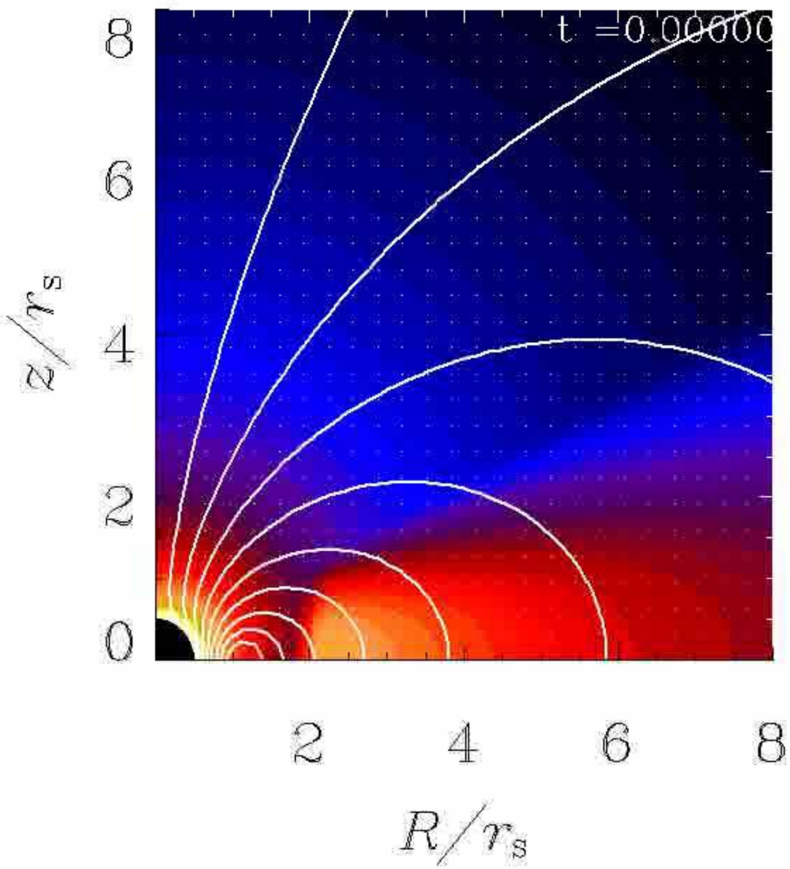
Solid white line:
Magnetic field line

Color: $\log \rho$

Numerical Result of Ideal GRMHD

Physical Review D **74**, 044005 (Aug., 2006)

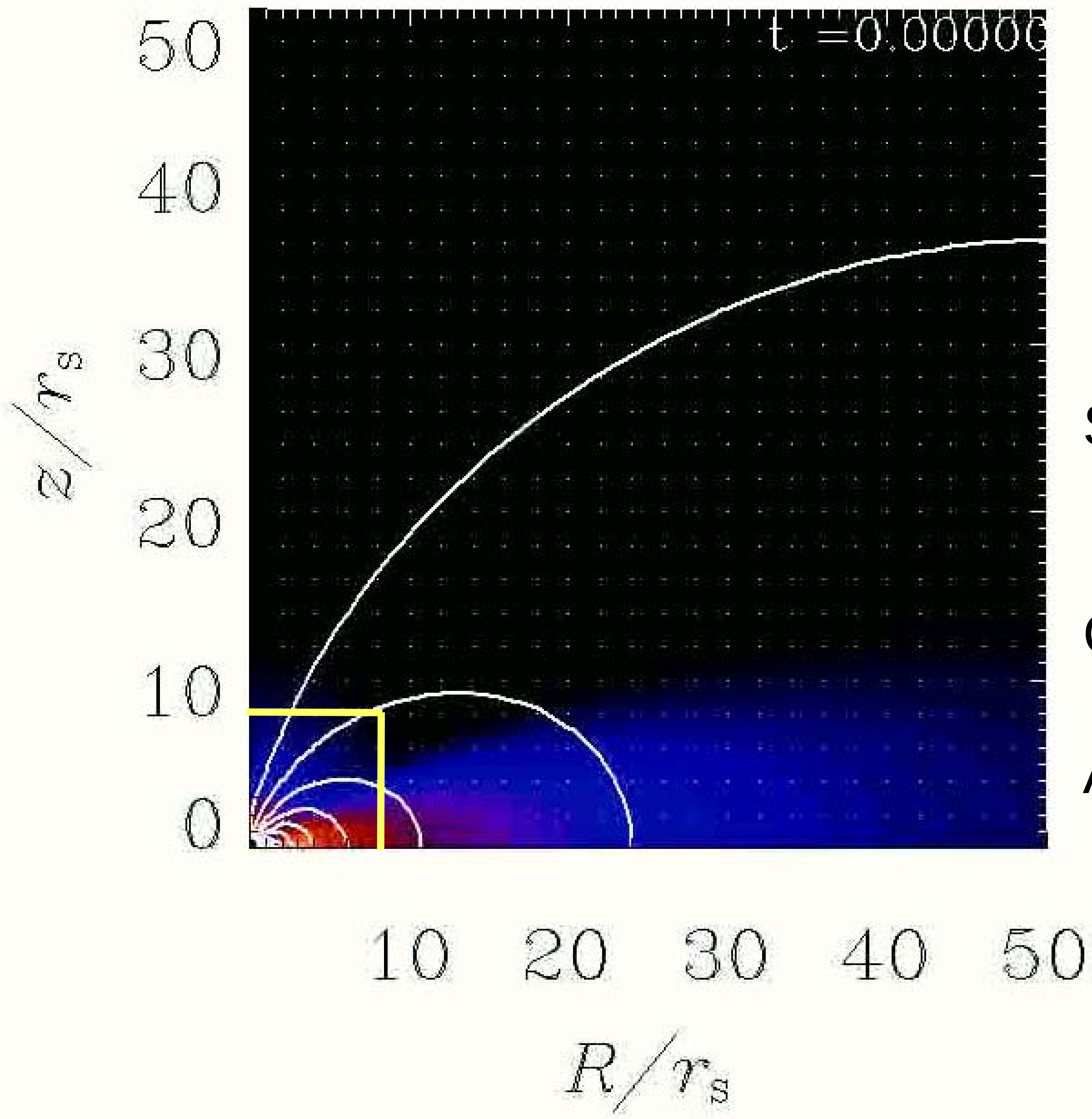
Time evolution:
Mass density, magnetic configuration



Solid white line:
Magnetic field surface

Color: $\log \rho$

Arrow: velocity



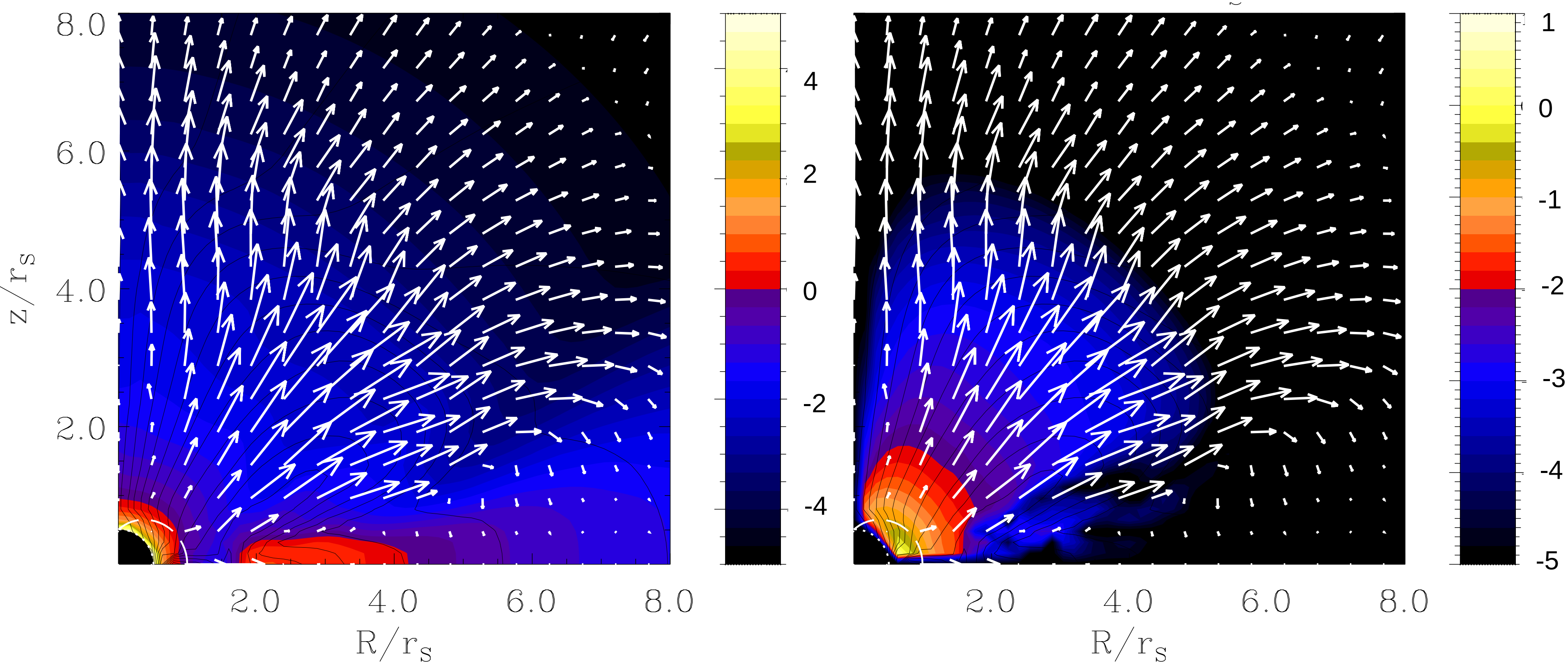
Solid line:
Magnetic field surface

Color: $\log \rho$

Arrow: Velocity

Mass density, velocity, magnetic pressure at $t = 20\tau_S$

$\log \rho$ Magnetic pressure, $\log\left(B_\phi^2 / 2\right)$



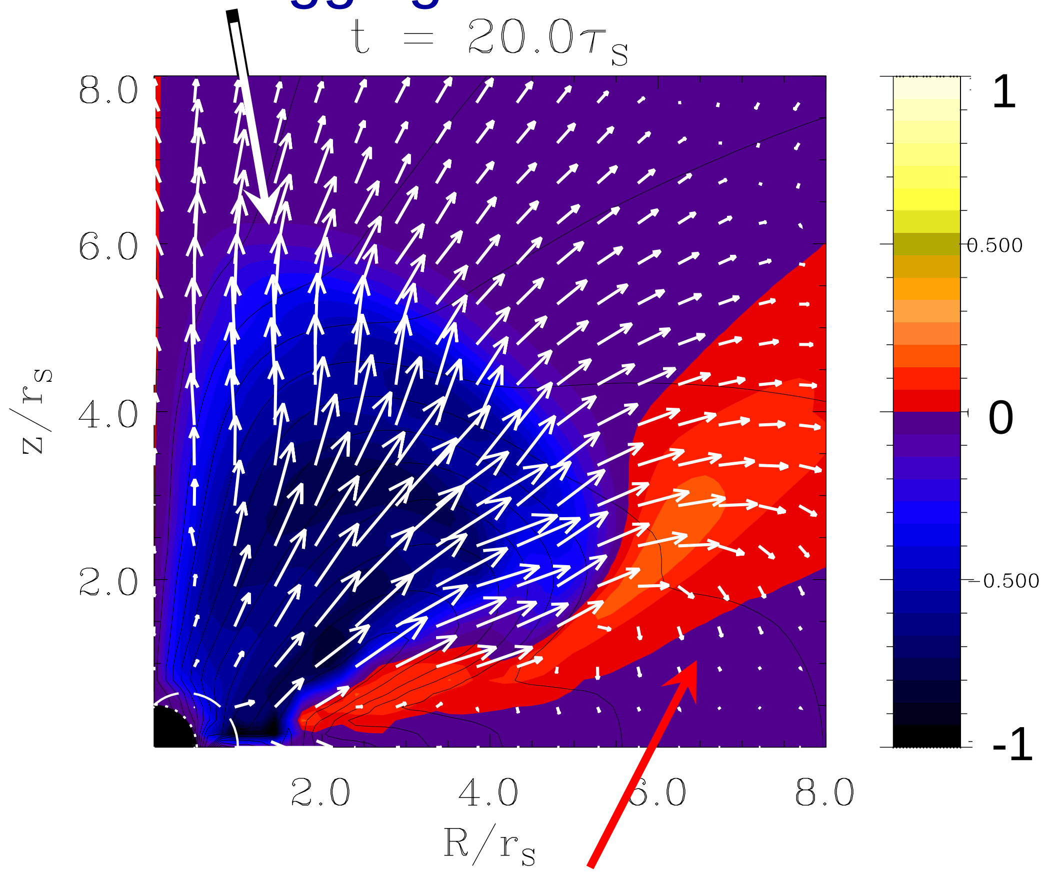
Solid line: Magnetic field line, Arrow: Velocity

v^{max} : 0.4c - 0.6c

$\tau_S = r_S / c$

Azimuthal component of magnetic field, magnetic configuration

Frame-dragging effect



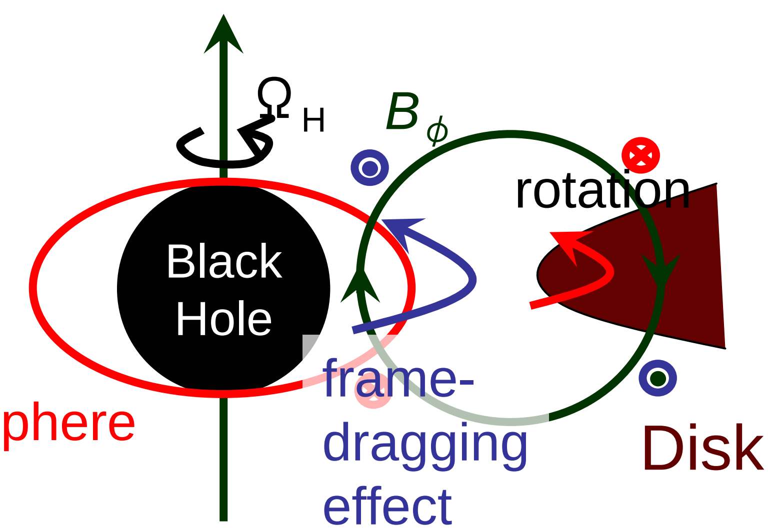
Disk rotation

$t = 20\tau_S$

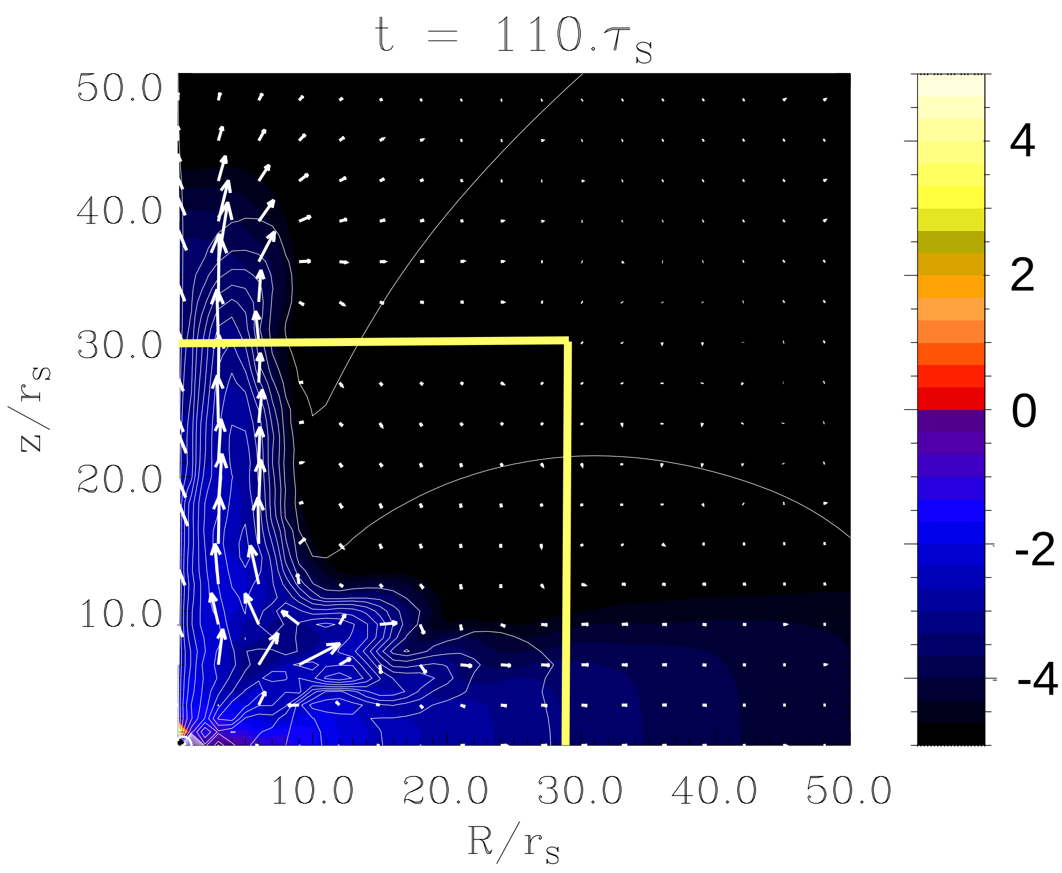
Solid line:
Magnetic field line

Color: $B_\phi / \rho^{1/2}$

Arrow: Velocity



Final stage of calculation：Density, velocity, magnetic configuration



$t = 110\tau_S$

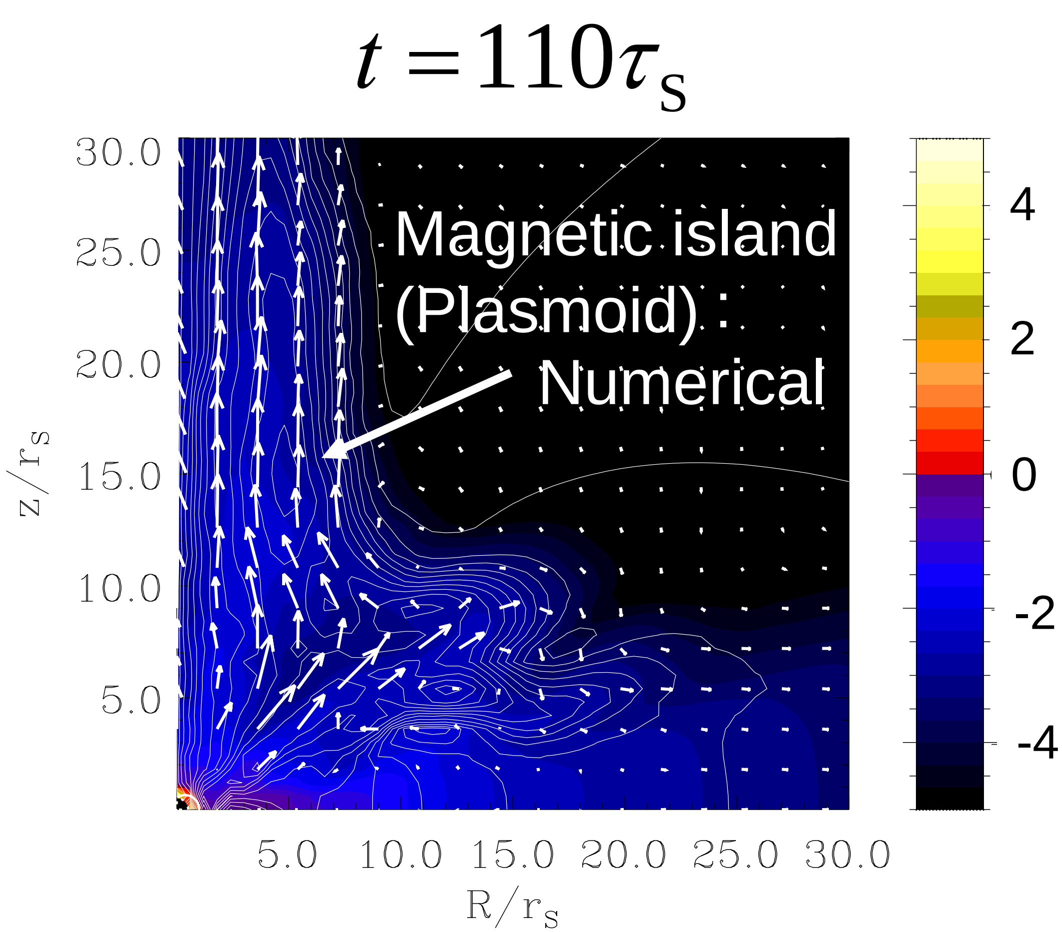
Solid line:
Magnetic field line

Color: $\log \rho$

Arrow: Velocity

v^{max} : 0.4*c* - 0.6*c*

Magnetic configuration of final stage: Numerical magnetic island

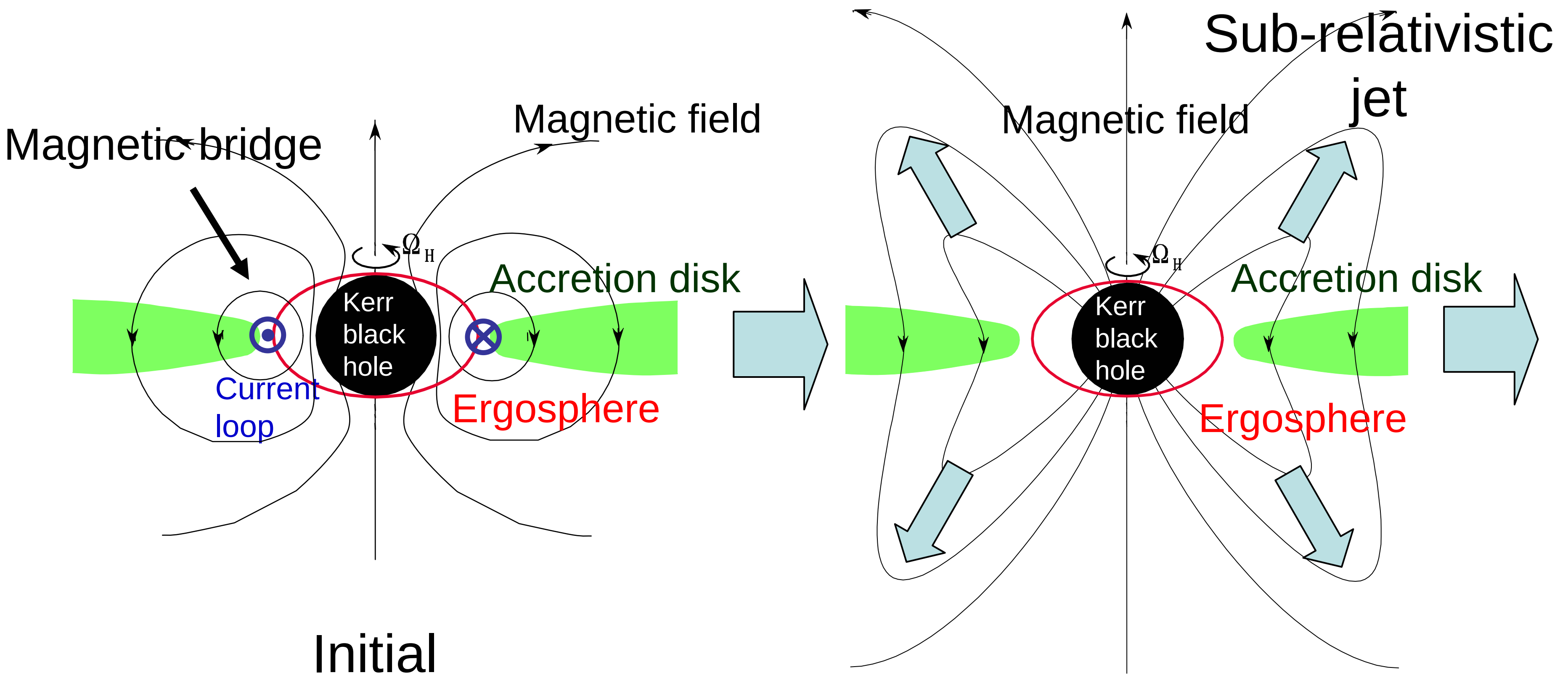


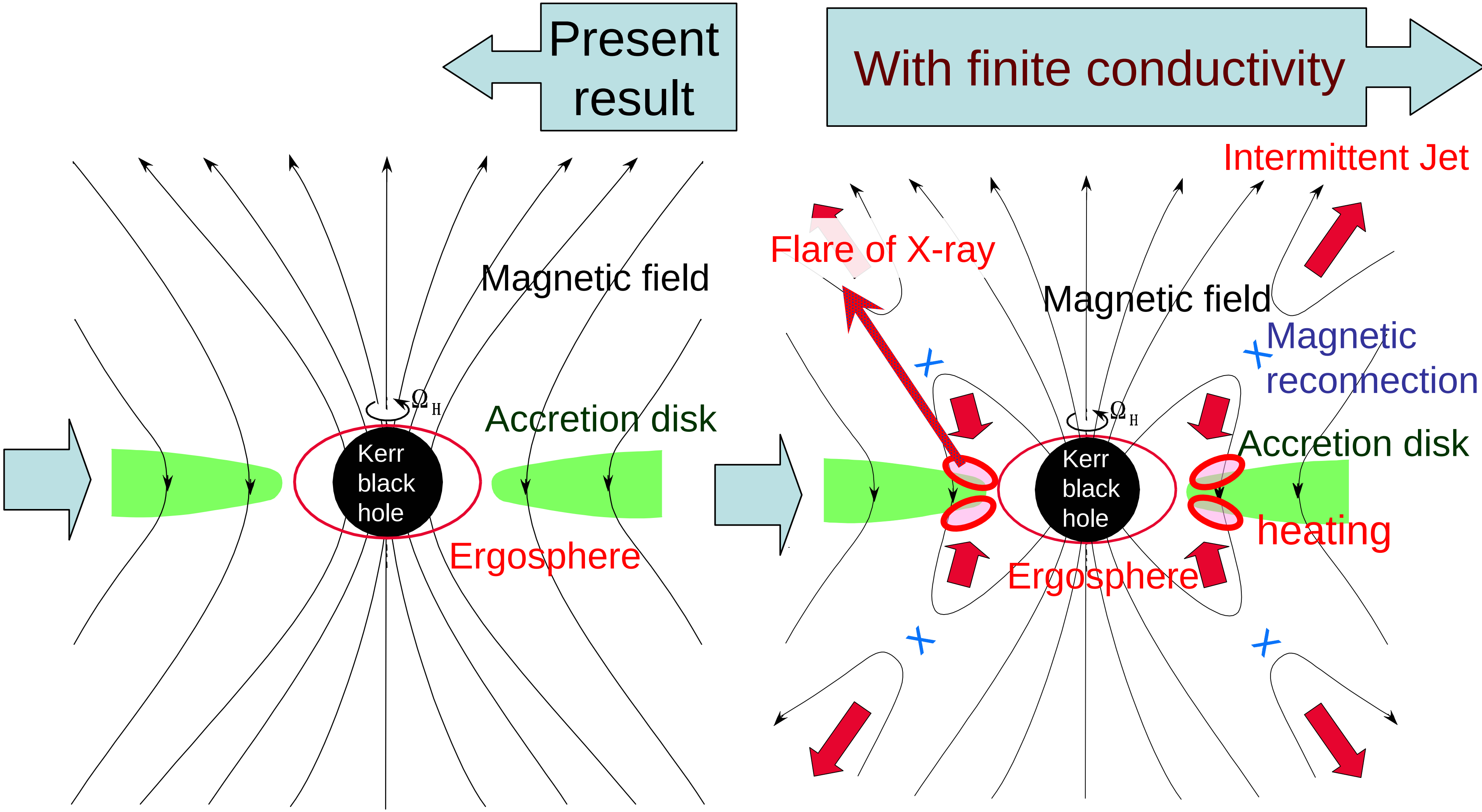
Ideal GRMHD:
No magnetic reconnection

- Magnetic Island: Numerical
- Anti-parallel magnetic field

Solid line:
Magnetic field line
Color: $\log \rho$
Arrow: Velocity

Schematic picture of phenomena caused by the magnetic bridge near the black hole





Anti-parallel magnetic field is formed

Magnetic reconnection must be important near black hole horizon!

GRMHD with finite conductivity

Near future important subject

Development of
GRMHD with finite conductivity

Present status:

- Numerical method
- Results of tests

GRMHD Equations with Finite Conductivity (σ GRMHD)

Special relativistic mass density, γρ

$$\frac{\partial D}{\partial t} = -\nabla \cdot [\alpha D(\mathbf{v} + c\boldsymbol{\beta})]$$

(conservation of particle number)

general relativistic effect

Special relativistic total momentum density

$$\frac{\partial \mathbf{P}}{\partial t} = -\nabla \cdot [\alpha(\mathbf{T} + c\beta\mathbf{P})] - \left(D + \frac{\varepsilon}{c^2}\right)\nabla(c^2\alpha) + \alpha\mathbf{f}_{\mathrm{curv}} - \mathbf{P} : \boldsymbol{\sigma}$$

(equation of motion)

special relativistic effect

Special relativistic total energy density

$$\frac{\partial \varepsilon}{\partial t} = -\nabla \cdot [\alpha(c^2\mathbf{P} - Dc^2\mathbf{v} + ec\boldsymbol{\beta})] - (\nabla\alpha)\cdot c^2\mathbf{P} - \mathbf{T} : \boldsymbol{\sigma}$$

(equation of energy)

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times [\alpha(\mathbf{E} - c\boldsymbol{\beta} \times \mathbf{B})]$$

$$\alpha(\mathbf{J} + \rho_e c\boldsymbol{\beta}) + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \left[\alpha\left(\mathbf{B} + \frac{\boldsymbol{\beta}}{c} \times \mathbf{E}\right)\right]$$

(Maxwell equations)

$$\nabla \cdot \mathbf{B} = 0$$

$$\rho_e = \frac{\alpha}{c^2}\nabla \cdot \mathbf{E}$$

no correspondence to non-relativistic MHD

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \frac{1}{\sigma\gamma}\left[\mathbf{J} - \gamma^2\left(\rho_e - \frac{1}{c^2}(\mathbf{v}\cdot\mathbf{J})\right)\mathbf{v}\right]$$

(Ohm's law with finite conductivity)

conductivity

New parts of σ GRMHD Equations

～ N. Watanabe & T. Yokoyama, astro-ph/0607285

Maxwell Equations

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left[\alpha(\mathbf{E} - c\boldsymbol{\beta} \times \mathbf{B})\right]$$

$$\frac{\partial}{\partial t}\left(\frac{\mathbf{E}}{c^2}\right) = \nabla \times \left[\alpha\left(\mathbf{B} + \frac{\boldsymbol{\beta}}{c} \times \mathbf{E}\right)\right] - \alpha\left(\mathbf{J} + \rho_\mathrm{e} c\boldsymbol{\beta}\right)$$

$$\frac{\partial \rho_\mathrm{e}}{\partial t} = -\nabla \cdot \left[\alpha\left(\mathbf{J} + \rho_\mathrm{e} c\boldsymbol{\beta}\right)\right]$$

(Conservation of electric charge)

Additional time evolutional Equations for σ GRMHD

$$\rho_\mathrm{e} = \frac{\alpha}{c^2} \nabla \cdot \mathbf{E}$$

Ohm's law with finite conductivity

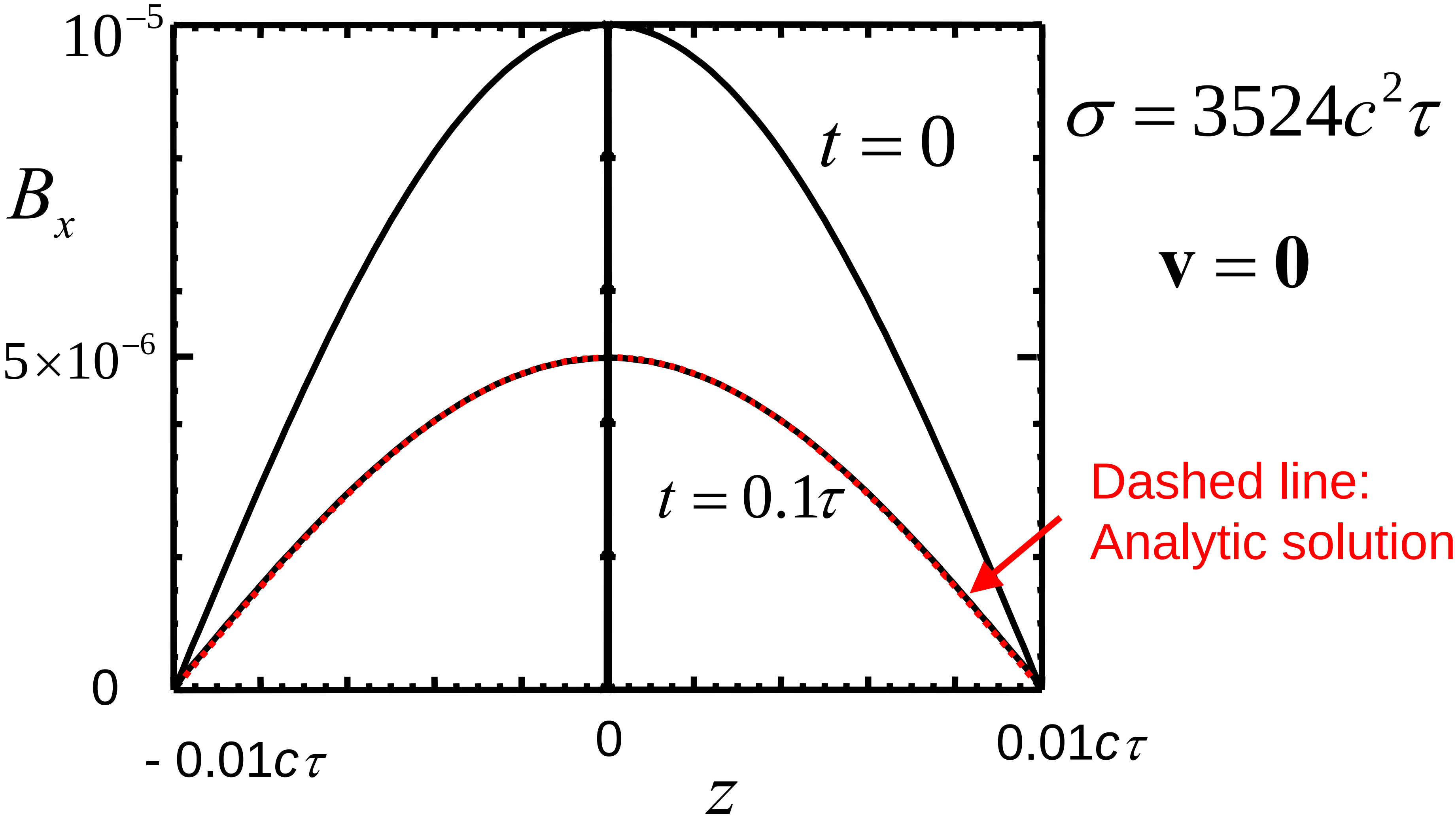
$$\mathbf{J} = \sigma\gamma\left[\mathbf{E} + \mathbf{v} \times \mathbf{B} - \frac{1}{c^2}(\mathbf{v} \cdot \mathbf{E})\mathbf{v}\right] + \rho_\mathrm{e}\mathbf{v}$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \frac{1}{\sigma\gamma}\left[\mathbf{J} - \gamma^2\left(\rho_\mathrm{e} - \frac{1}{c^2}(\mathbf{v} \cdot \mathbf{J})\right)\mathbf{v}\right]$$

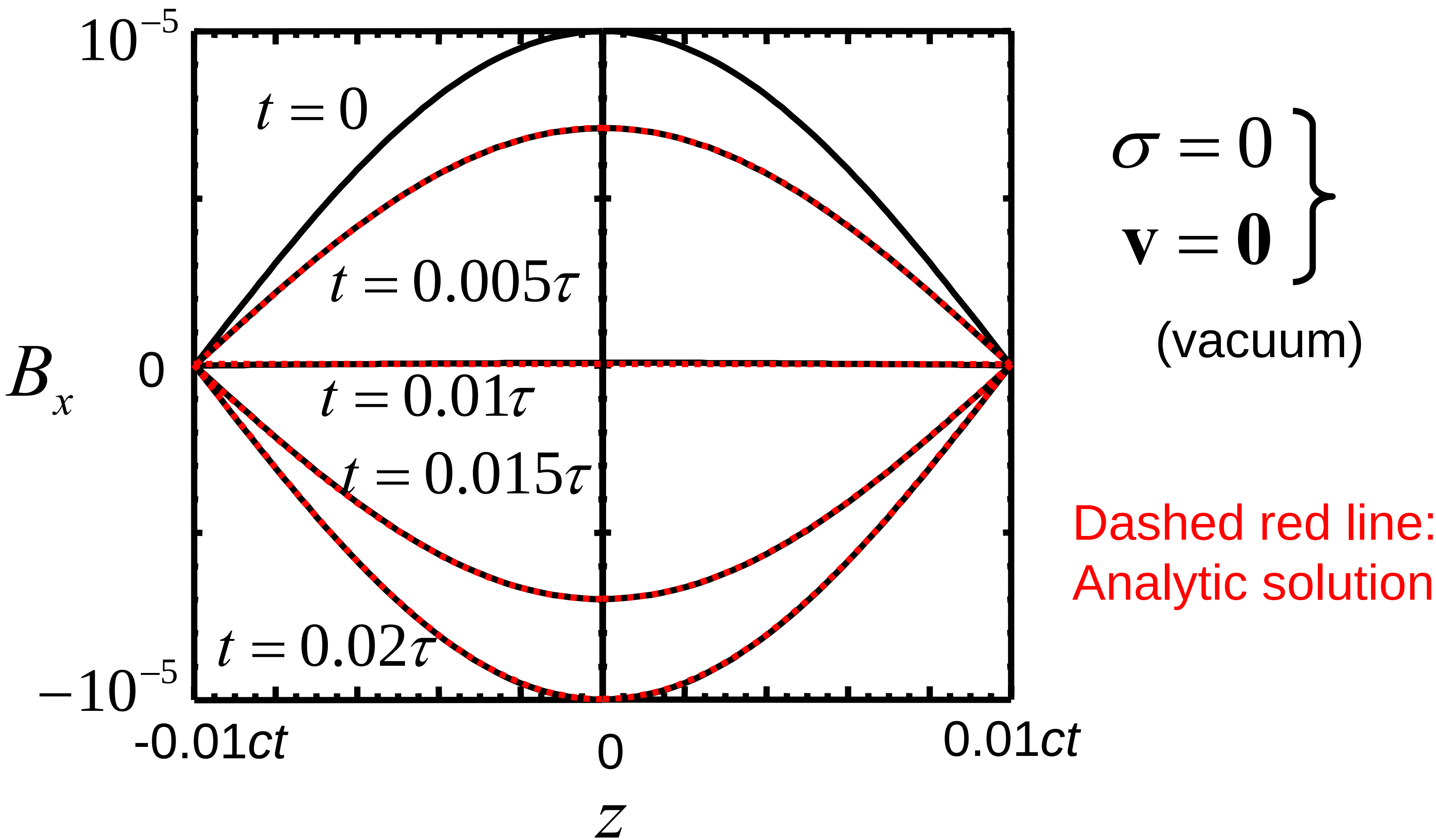
σ GRMHD: tests

- Magnetic diffusion
- Wave propagation
- Recalculation of dynamics of magnetic bridge with large conductivity

Test of σ GRMHD with magnetic diffusion

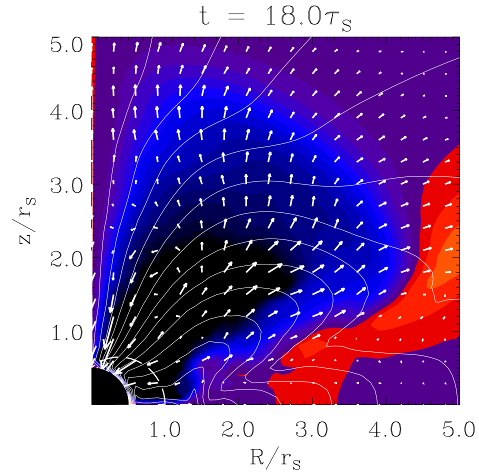


Test of σ GRMHD with Electromagnetic standing wave in vacuum



Test : Recalculation of Dynamics of Magnetic Bridge with finite (large) σ GRMHD

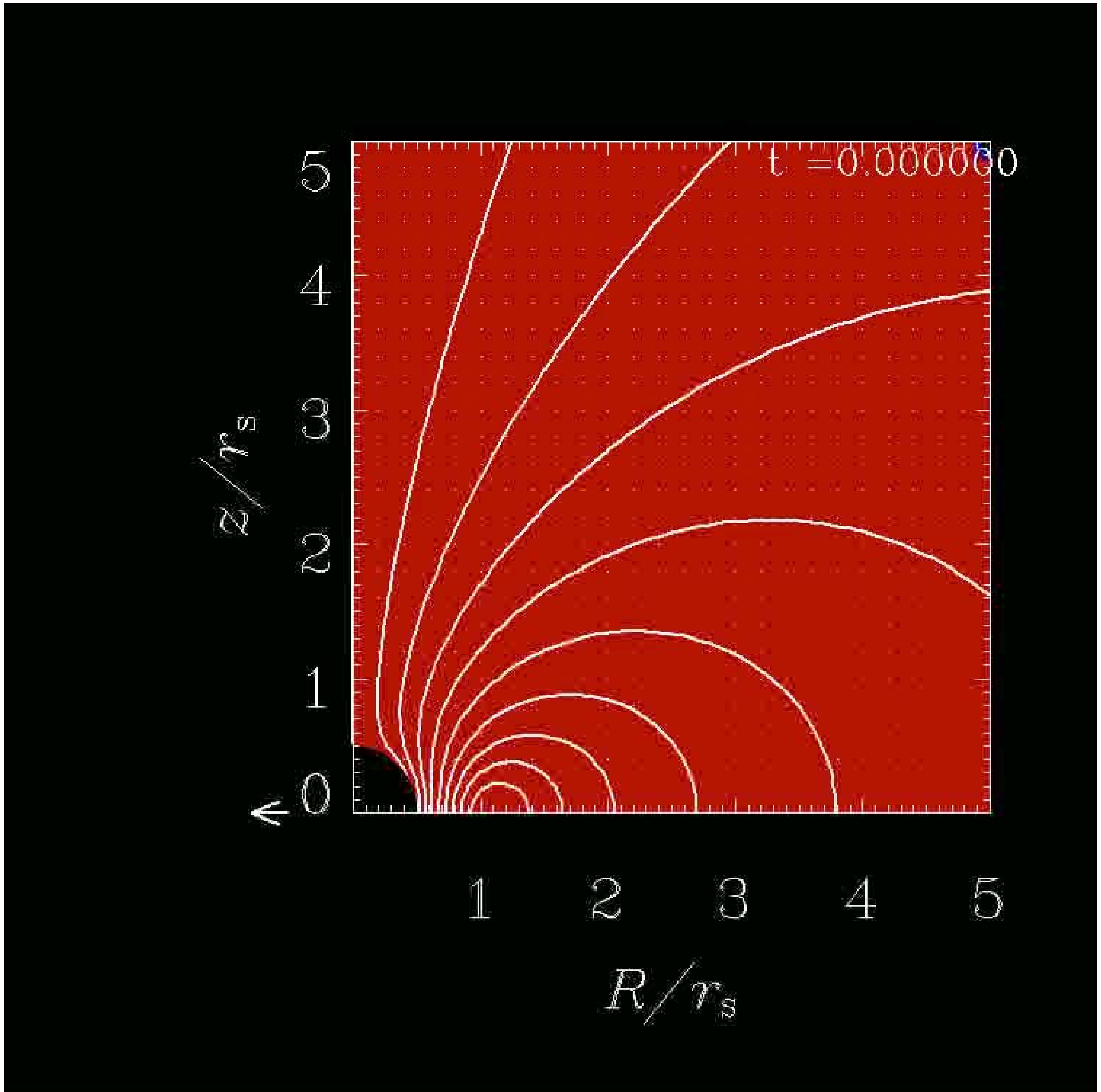
- $\sigma \gg 1/(r_S c) \Rightarrow$
  Results similar to that of ideal GRMHD



Here, we compare the results of ideal and finite (large) σ GRMHD simulations of magnetic bridges between the ergosphere and the disk around black hole (corona: quasi-equilibrium).

Quasi-equilibrium: $r \rightarrow r_H$ plasma falling toward black hole
$r \rightarrow \infty$ plasma in hydrostatic equilibrium

Ideal GRMHD simulation of magnetic bridge in quasi-equilibrium corona

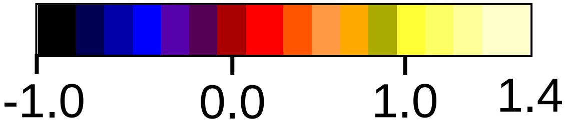


$\sigma = \infty$

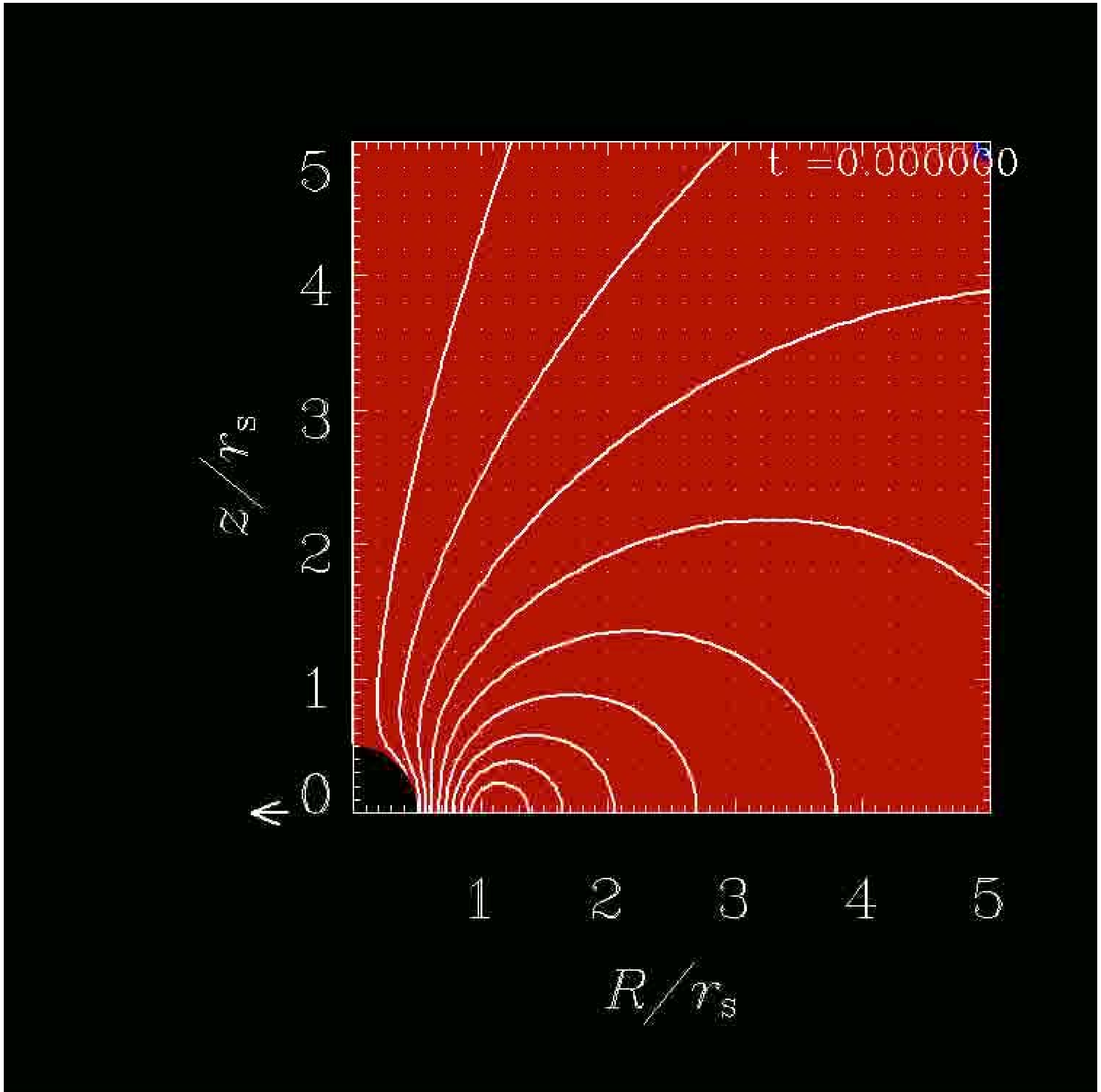
Solid line:
Magnetic field line

Color: $B_\phi / \rho^{1/2}$

Arrow: Velocity



Finite σ GRMHD simulation of magnetic bridge in quasi-equilibrium corona

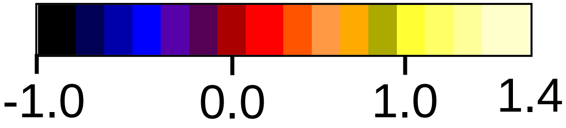


$\sigma = 100 / c r_S$

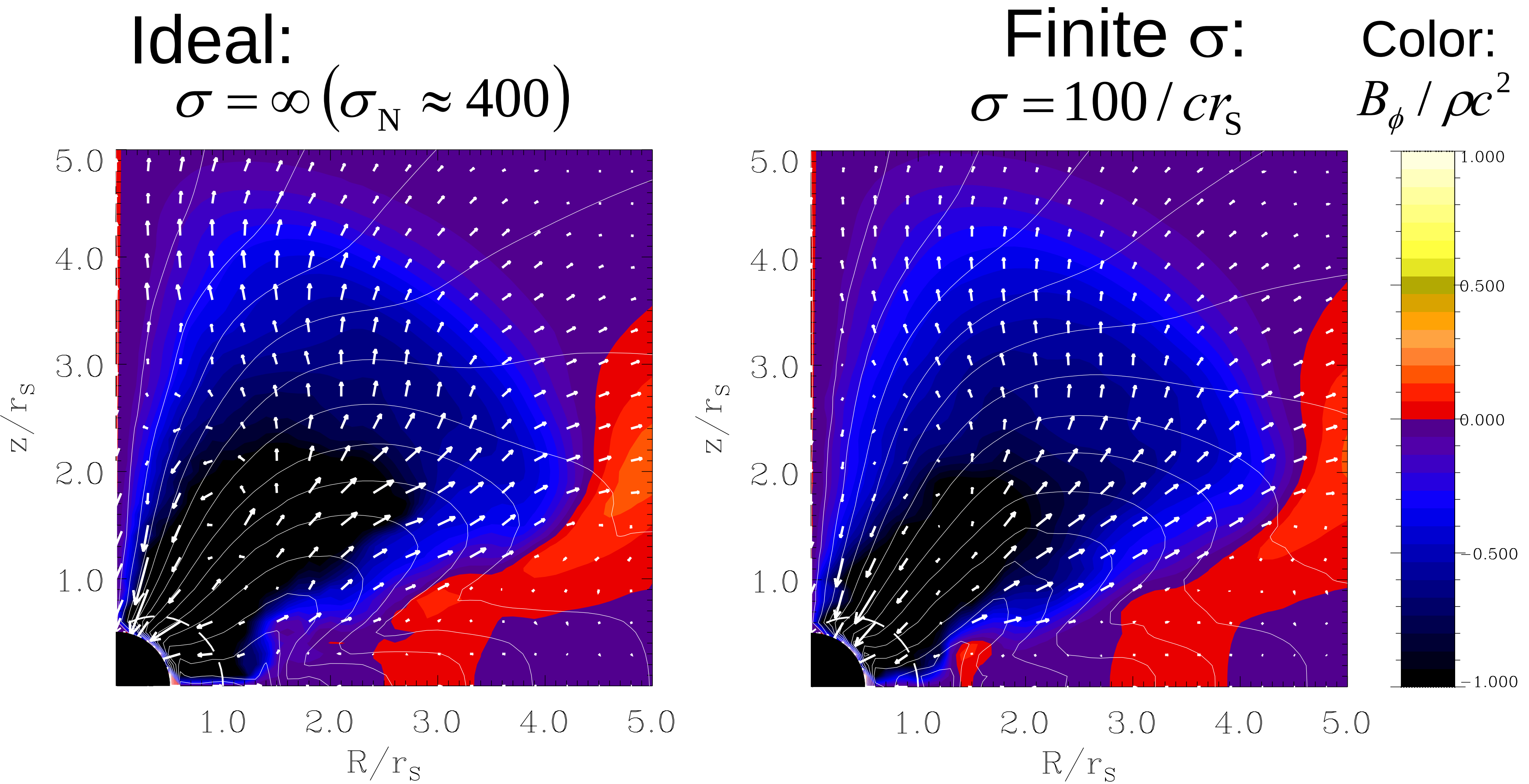
Solid line:
Magnetic field line

Color: $B_\phi / \rho^{1/2}$

Arrow: Velocity



Comparison of results of ideal and finite σ GRMHD simulations at $t = 18\tau_S$ (no anti-parallel magnetic field)



Solid line: Magnetic field line, Arrow: Velocity

$\tau_S = r_S / c$

Summary

- Ideal GRMHD numerical results show that the magnetic bridges between the ergosphere and disk around rapidly rotating black hole can not be stationary and expand explosively to form a jet or it is swallowed by the black hole.
- The anti-parallel magnetic field is formed in the jet where the magnetic reconnection will take place, which may influence the jet propagation.
- To investigate the magnetic reconnection, GRMHD with finite conductivity (σGRMHD) is required. We showed the basic numerical method of σGRMHD and test calculations for it.

Near future plan

- Further test calculations of σ GRMHD in 1-D:
  - – with other kinds of electromagnetic, plasma waves
- Test calculations of σ GRMHD in 2-D:
  special relativistic/nonrelativistic magnetic reconnection
  - – with tearing instability
  - – with reconnection with anomalous resistivity
- Long-term σ GRMHD simulations of relativistic jet formation by magnetic bridge around black hole with finite (anomalous?) conductivity:

⇨ the significant role of the magnetic reconnection in the jet formation with magnetic bridge.