Instability of Pedestrian Flow in 2D Optimal Velocity Model with Attractive Interaction

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CONTENTS

Introduction

Pedestrian flow

- Social / Engineering Interest
  Ship or Building evacuation
- Physical Interest
  To understand pedestrian behavior

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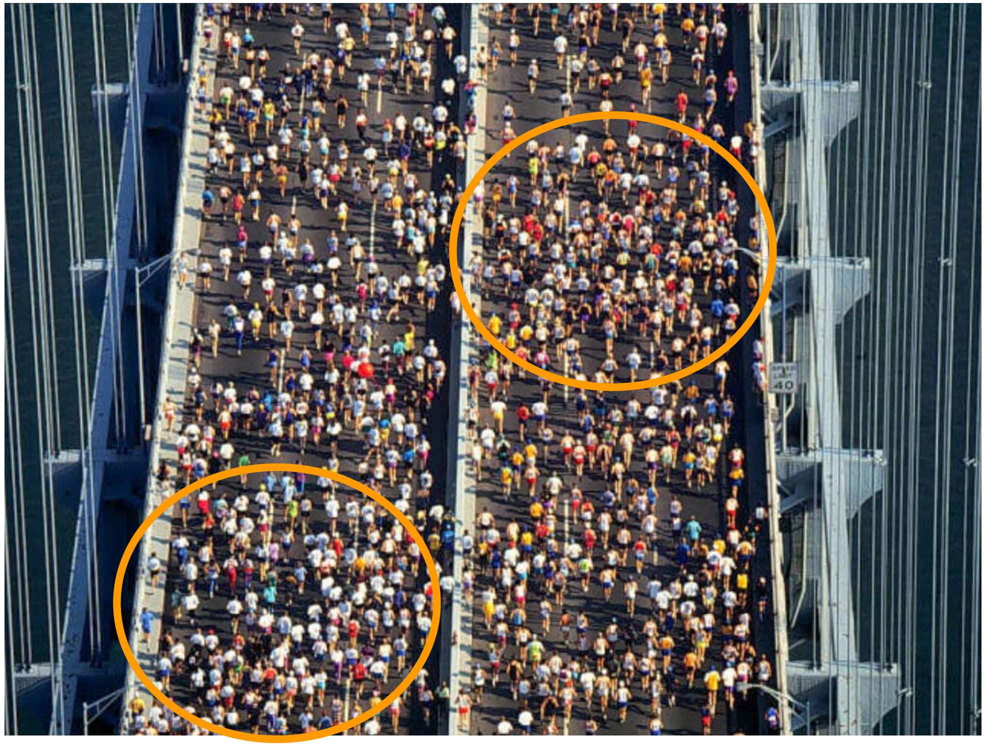
Dynamical model

Fluctuation? →



trafffic jam?

↓

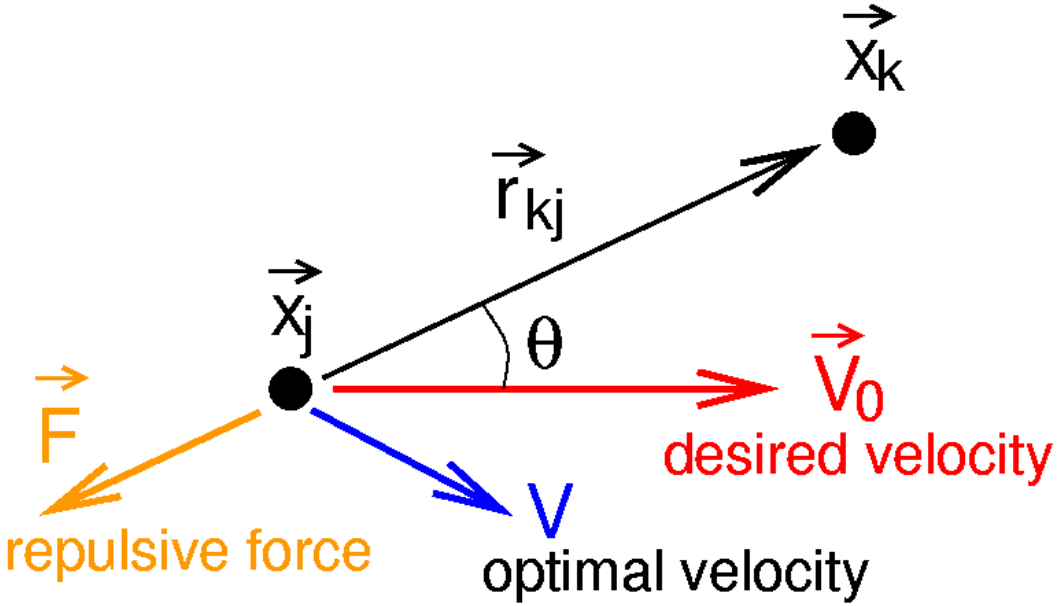
Optimal Velocity (OV) model

2-dim OV Model

$$\frac{d^2}{dt^2}\vec{x}_j(t) = a\left[\underline{\left\{\vec{V}_0 + \sum_k \vec{F}(\vec{r}_{kj(t)})\right\}} - \frac{d}{dt}\vec{x}_j(t)\right]$$

$\vec{V}_0$: desired velocity

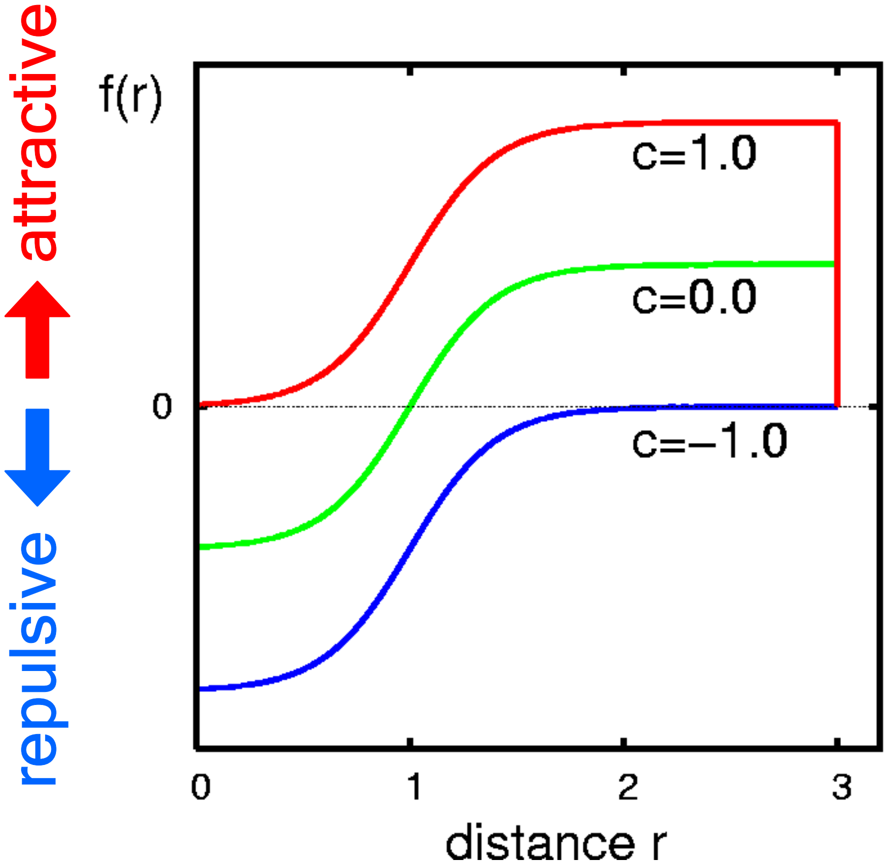
$\vec{F}(\vec{r}_{kj})$: repulsive / attractive interaction

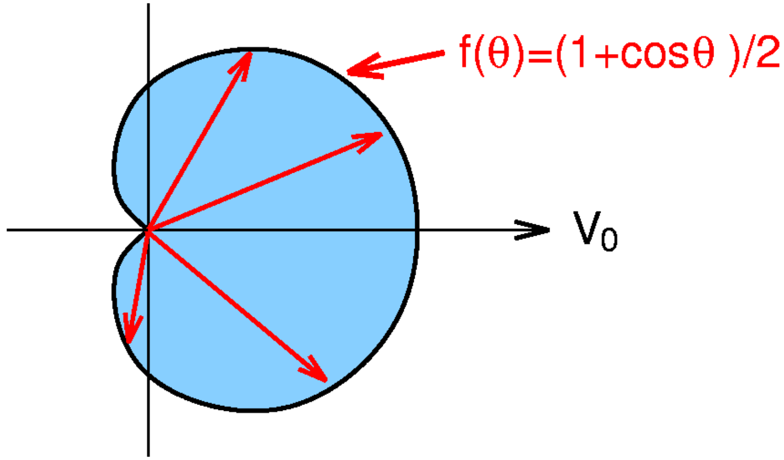


OV function expresses repulsive / attractive interaction

$$\vec{F}(\vec{r}_{kj}) = \underbrace{\frac{1}{2}\left[\tanh \beta(r_{kj} - b) + c\right]}_{\text{distance dependence}} \underbrace{\frac{(1 + \cos\theta)}{2}}_{\text{angular dependence}} \vec{n}_{kj}$$

distance dependence   angular dependence

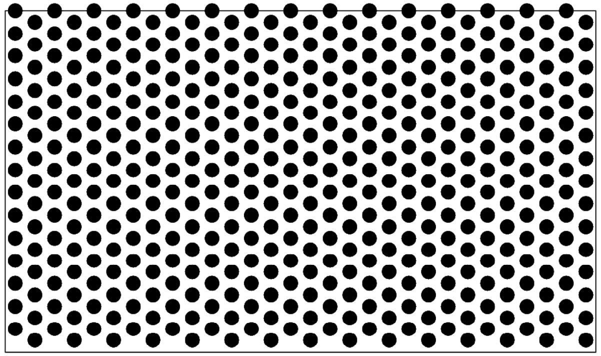




$$\vec{n}_{kj} = \frac{\vec{r}_{kj}}{r_{kj}}, \quad r_{kj} = |\vec{r}_{kj}|$$

Stability Condition of Homogeneous Flow

homogeneous flow



direction of motion

Linear analysis

mode solution $$\begin{pmatrix} x_j \\ y_j \end{pmatrix} = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} \exp[i\omega t + i\vec{k} \cdot \vec{x}]$$

Three types of modes

(a) $\begin{pmatrix} \epsilon \\ 0 \end{pmatrix}$ ~ longitudinal mode along X-axis

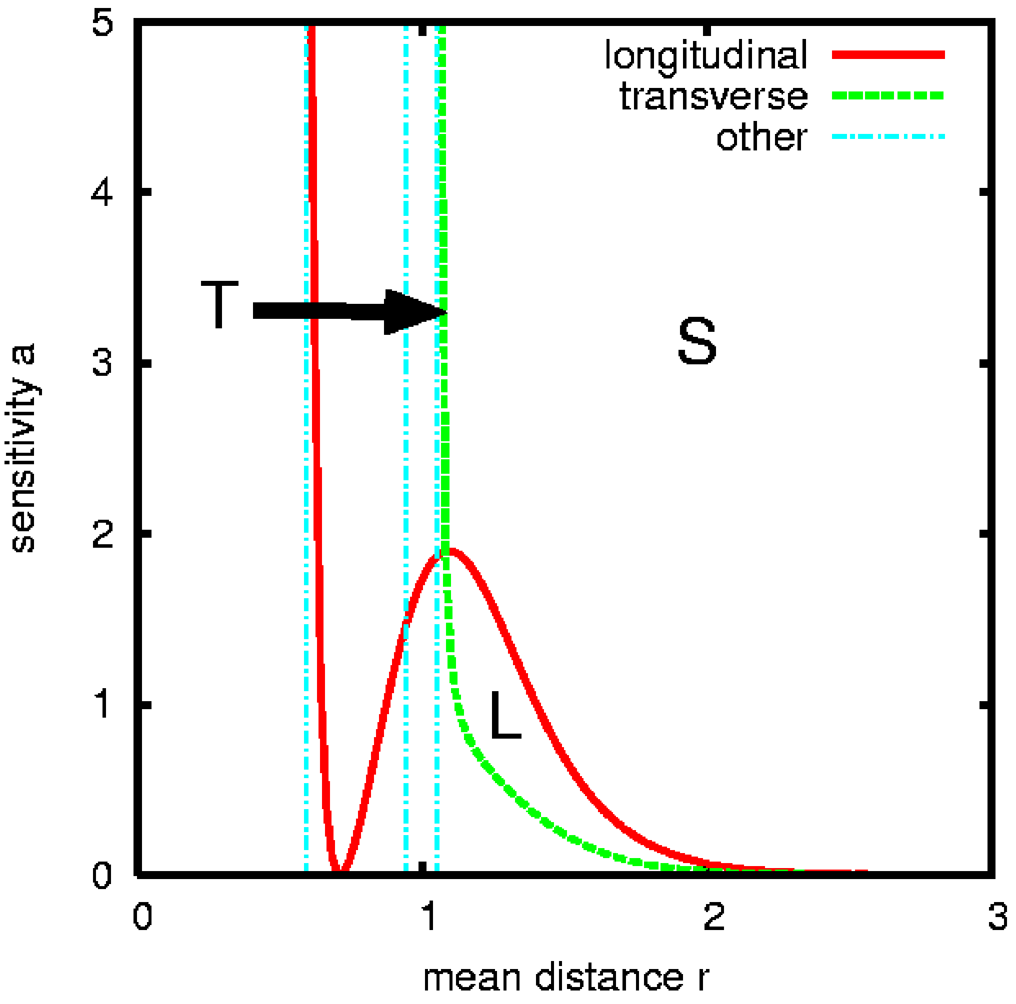
(b) $\begin{pmatrix} 0 \\ \epsilon \end{pmatrix}$ ~ transverse mode along X-axis

(c) $\begin{pmatrix} 1 \\ \epsilon \end{pmatrix}$ elliptically polarized mode

Skip calculations. They are very long and complicated.

Phase Diagram & Simulations

c=-1 : repulsive interaction only



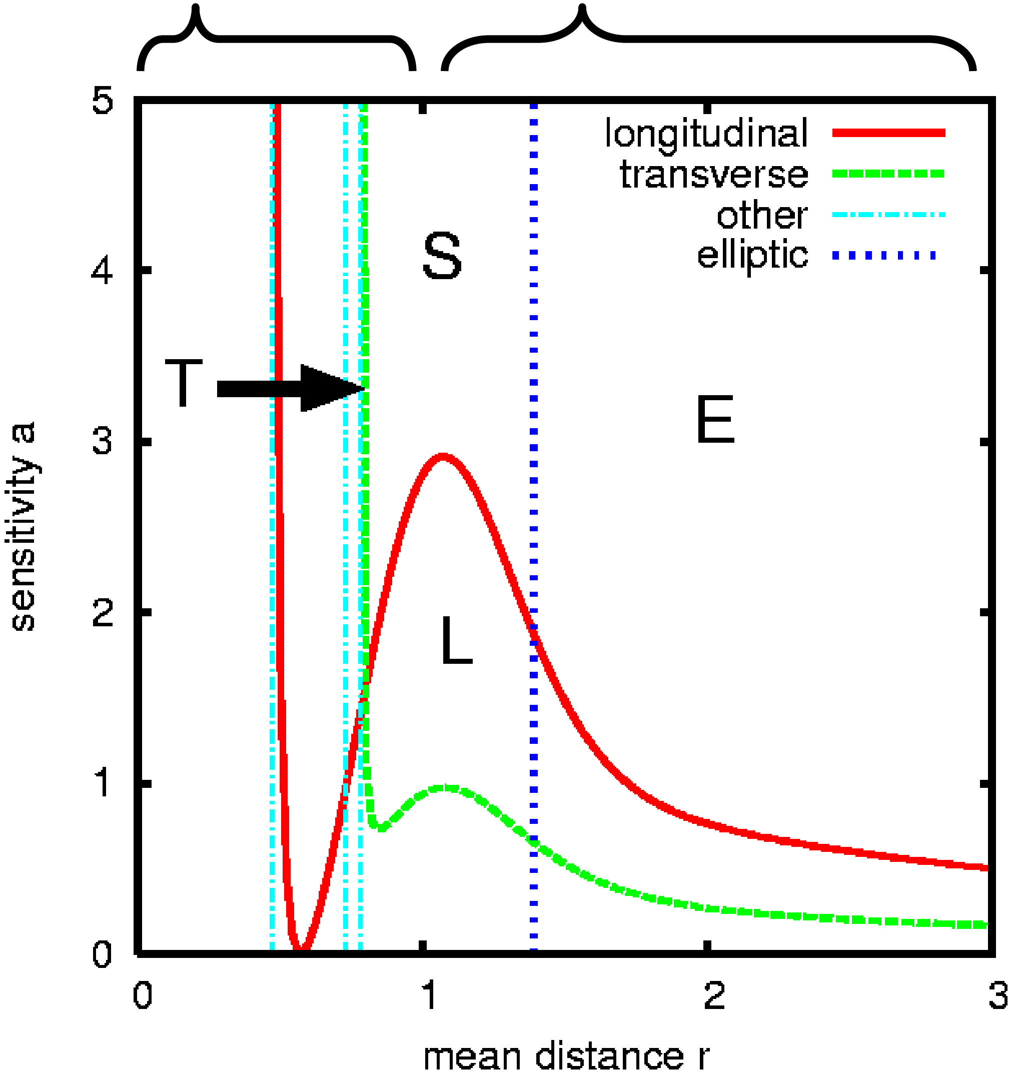
Stable homogeneous flow

Transverse mode is unstable

Longitudinal mode is unstable

Phase Diagram & Simulations

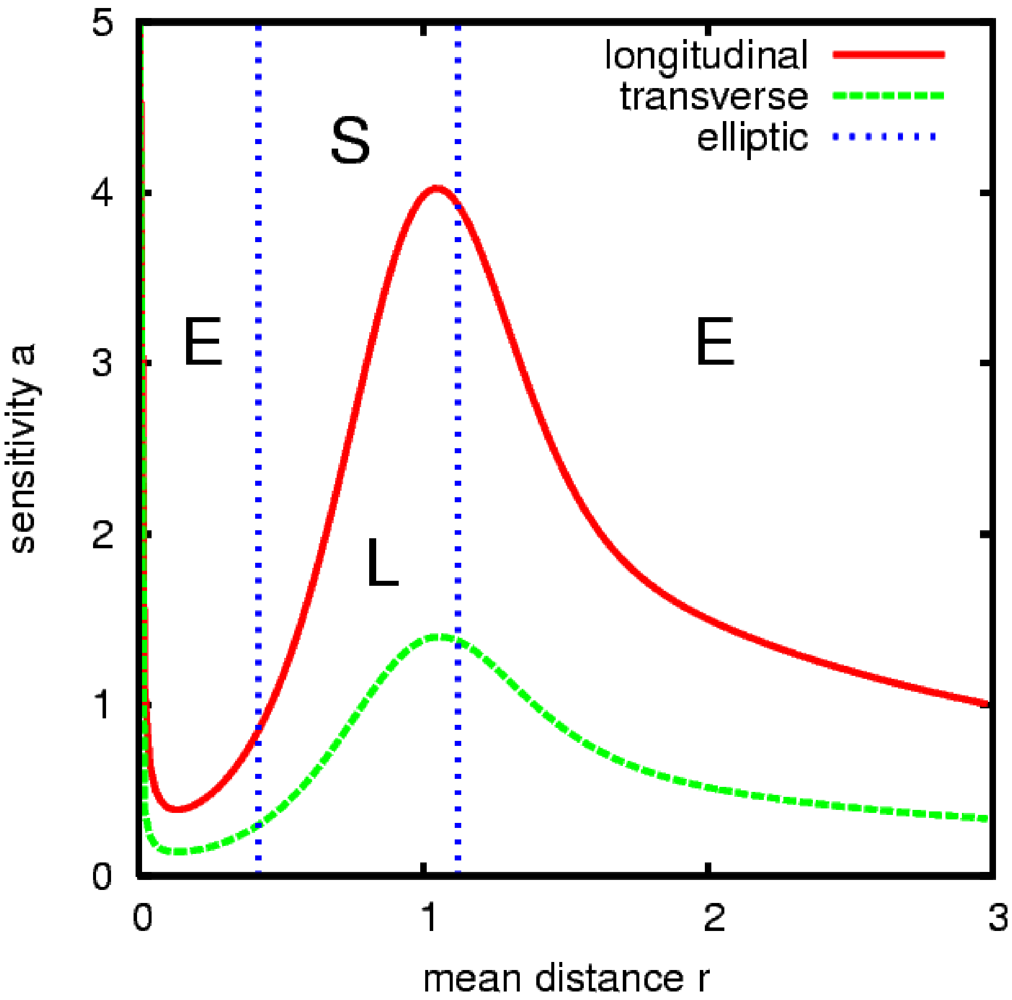
c=0 : both repulsive and attractive interaction



Elliptically polarized mode is unstable

Phase Diagram & Simulations

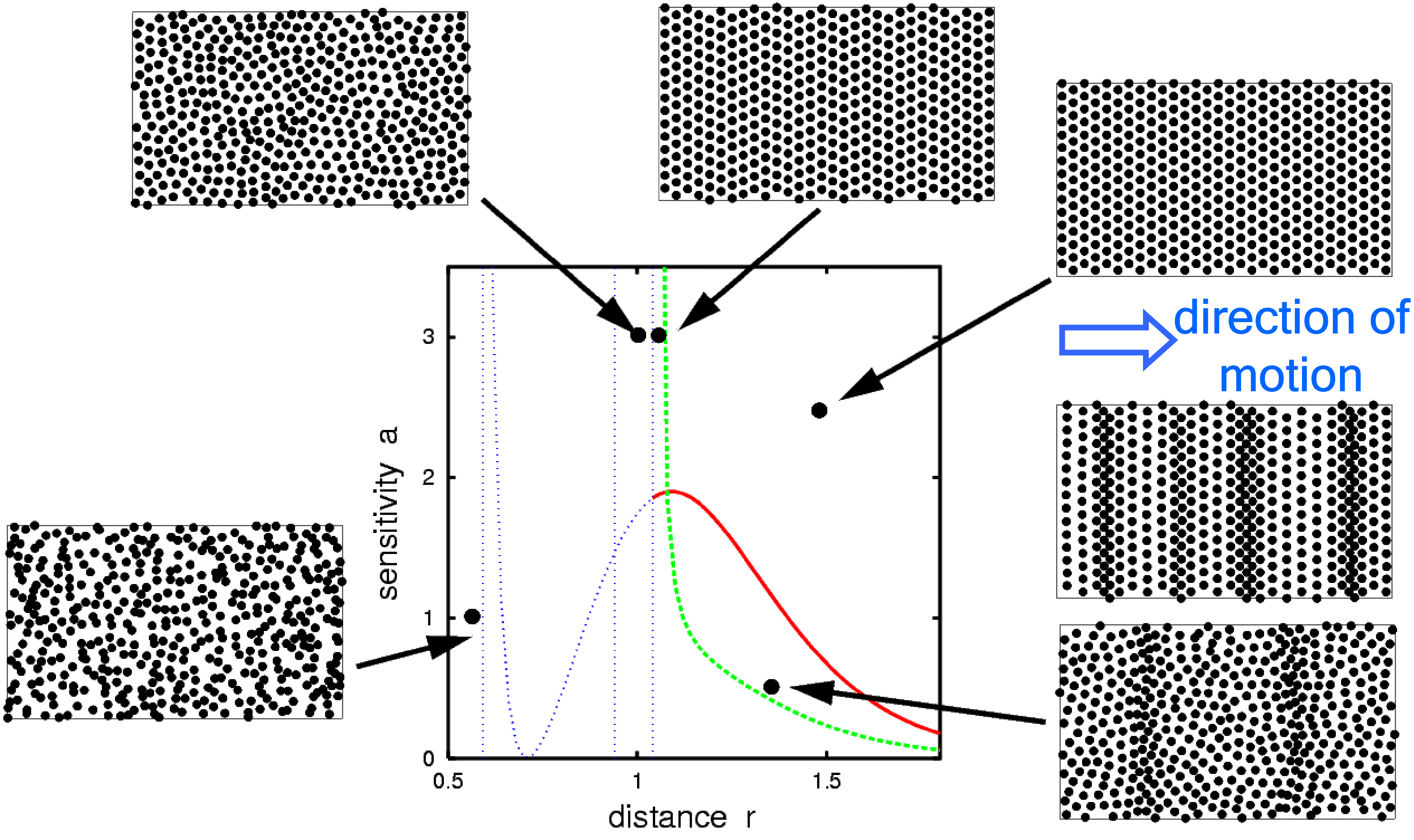
c=1 : attractive interaction only



Analogy to spring
"Stable region exists
around the neutral
point."
is not valid.

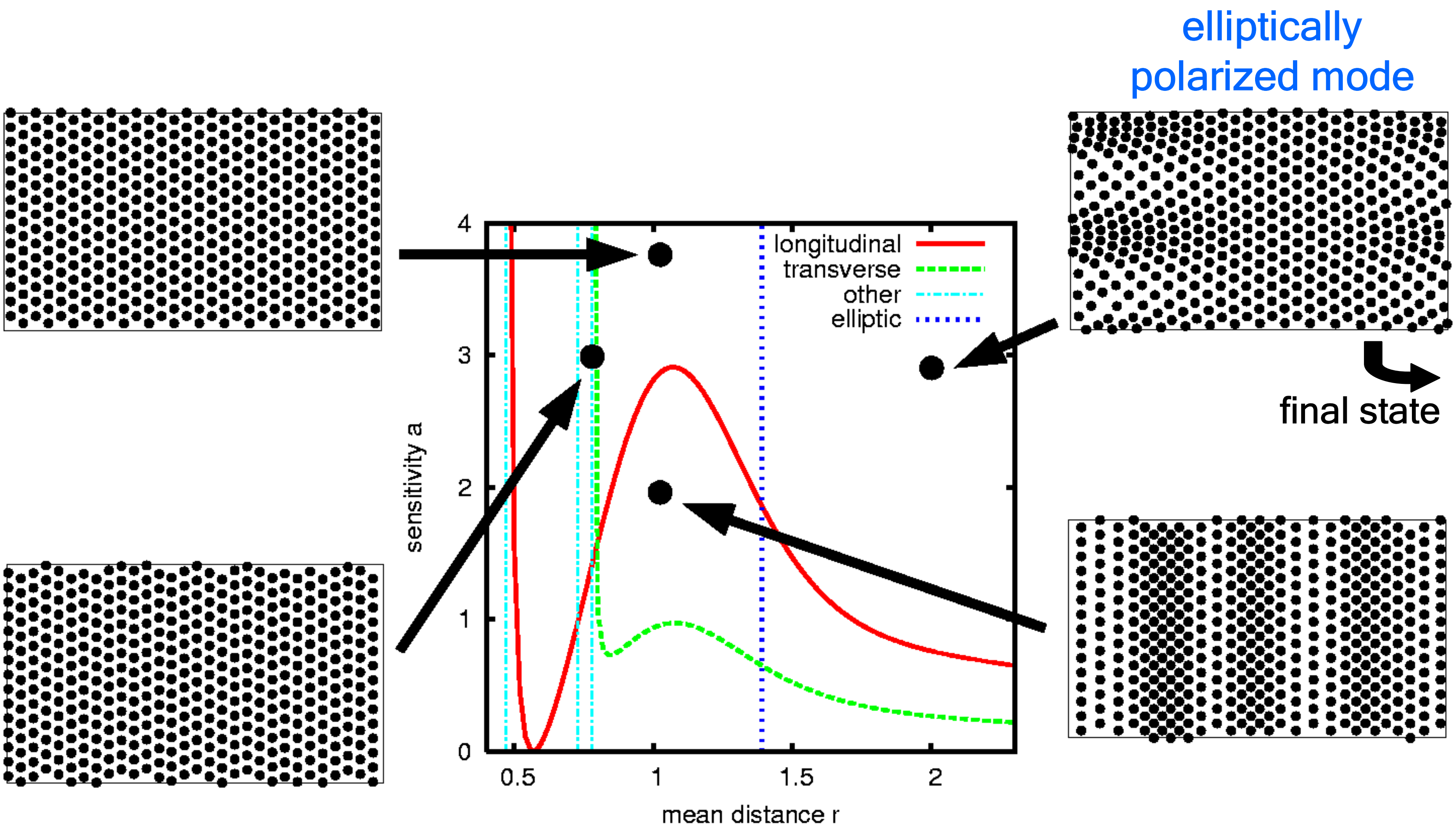
Correspondence between linear analysis and simulation

c=-1 : repulsive interaction only



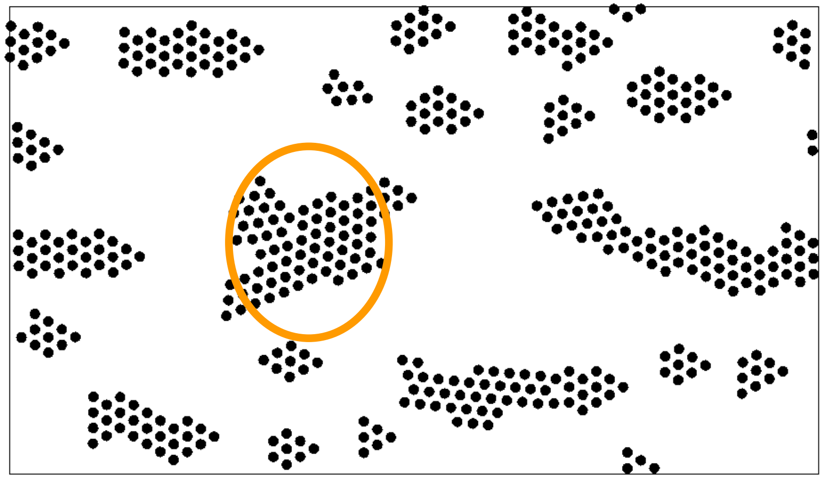
Correspondence between linear analysis and simulation

c=0 : both repulsive and attractive interaction



Final state of simulation

c=0



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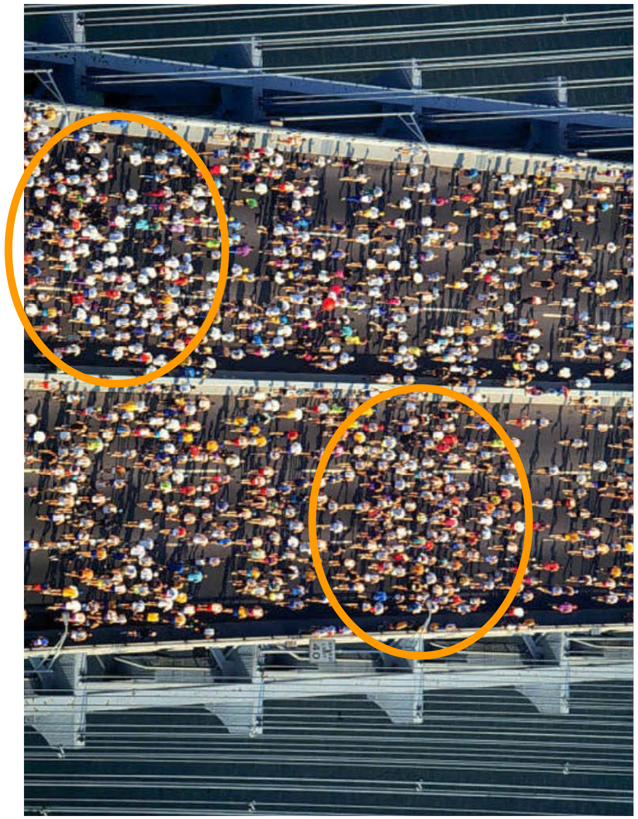
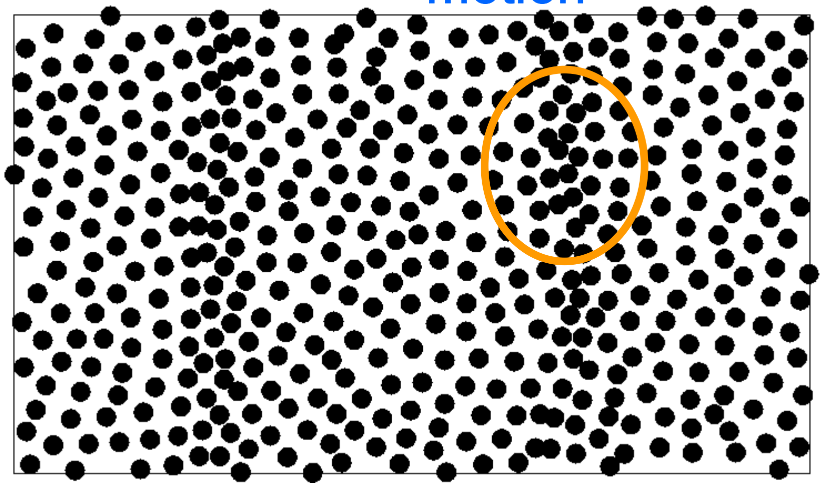


direction of motion

c=-1



Summary

- We obtain phase diagrams in 2D OVM with attractive interaction.
- Attractive interaction breaks sparse homogeneous flow, even if it is small.
- There is a new phase due to the instability of elliptically polarized modes.

Discussion

- Can collective bio-motion be understood in this framework?
- Final states depend on the detail of the model. What is the most realistic interaction? (Lenard-Jones?)
- 3D?

Reference
A.N, K.H and Y.S, Phy. Rev. E71, 036121, (2005).
A.N, K.H and Y.S, in preparation.