Partially Synchronized Neural Networks

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review of various papers by:

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Hilbert J. Kappen

- Dutch Foundation of Neural Networks (SNN), and Department of Medical Physics and Biophysics, University of Nijmegen

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Interesting features in a network:

- **Topology of connections**
  - fully connected (each node linked to all the others)
  - randomly diluted (some broken connections)
  - scale invariant connectivity, etc.
- **Weighted, and possibly time–varying communication lines**
  - food webs, ecological and metabolic nets (chains, fluxes different intensity)
  - Internet, WWW, economic and other social nets (agents interchange different amounts of information or money)
  - transport connections differ in capacity, number of flights and passengers
  - spin-glass and reaction-diffusion systems (diffusion of ions, local rearrangements and reactions vary the interactions between the units)
  - the central nervous system and the brain…
- **Nodes not fully, constantly synchronized to perform a task**
  - may be a matter of economy… and perhaps a must in some cases…
  - e.g., only fraction of neurons sometimes activated in a region at given time; rest no input but maintain memory of the previous state

Let network with

- neuron, spin, …processor,… at each node;
- set of node activities, $\boldsymbol{\sigma} \equiv \{\sigma_i\}$ (e.g. $\sigma_i = \pm 1$)
- communication line weights, $\boldsymbol{w} \equiv \{w_{ij} \in \mathbb{R}\}$ ($i,j = 1,\ldots,N$)
- local field $h_i(\boldsymbol{\sigma}, \boldsymbol{w})$ on node i induced by the weighted action of the other, $N - 1$ nodes
- additional, operational set of binary indexes,

$$\boldsymbol{x} = \{x_i = 0 \text{ or } 1\}$$

(to help in choosing from the set of N nodes)

Time evolution according to generalized cellular-automaton strategy:

- at each t, update $1 \le n \le N$ variables; evolves in discrete t:

$$P_{t+1}(\boldsymbol{\sigma}) = \sum_{\boldsymbol{\sigma}'} R\left(\boldsymbol{\sigma}' \to \boldsymbol{\sigma}\right) P_t(\boldsymbol{\sigma}')$$

- with transition rate:

$$R\left(\boldsymbol{\sigma} \to \boldsymbol{\sigma}'\right) = \sum_{\mathbf{x}} p_n(\mathbf{x}) \prod_{\{i|x_i=1\}} \tilde{\varphi}_n\left(\sigma_i \to \sigma_i'\right) \prod_{\{i|x_i=0\}} \delta_{\sigma_i,\sigma_i'}$$

$$\tilde{\varphi}_n\left(\sigma_i \to \sigma_i'\right) \equiv \varphi\left(\sigma_i \to \sigma_i'\right)\left[1 + \left(\delta_{\sigma_i',-\sigma_i} - 1\right)\delta_{n,1}\right]$$

- $\varphi = f(\beta \sigma_i h_i)$, β inverse "temperature"; choose the n sites at random, i.e.,

$$p_n(\mathbf{x}) = \begin{pmatrix} N \\ n \end{pmatrix}^{-1} \delta\left(\sum_i x_i - n\right)$$

Is natural generalization of two familiar cases:

- $n = 1$, **$\rho \equiv n/N \to 0$: <u>sequential</u> (Glauber) updating**
- $n = N$, **$\rho \to 1$: <u>parallel</u> (Little, automata) updating**

$\rho \in (0,1)$: <u>crossover</u> between these two situations

Assuming a cell stimulated only in the presence of a neuromodulator such as dopamine, n will correspond to the number of neurons that are modulated each cycle; the other, $N - n$ neurons would receive no input to maintain memory of the previous state:

mimics persistent neural activity claimed to be at the basis of working memories, e.g., A.V. Egorov et al., *Nature* <u>420</u> (2000)

Marro, Torres, Cortes & Wemmenhove, *PRL* 2006, submitted.

Local fields are defined:

$$h_i\left(\boldsymbol{\sigma}, \mathbf{w}\right) = \sum\nolimits_{j \neq i} w_{ij} \sigma_j$$

where the communication line weights w_{ij} depend on situation of interest.

Here, we remark:

(Confession: we wish the fields to simulate "depression" of synapses associated to activity–dependent synaptic *noise*)

- Short–time **fluctuations of synapses** occur that compete with other mechanisms during transmission of information, and ultimately determine processing of information

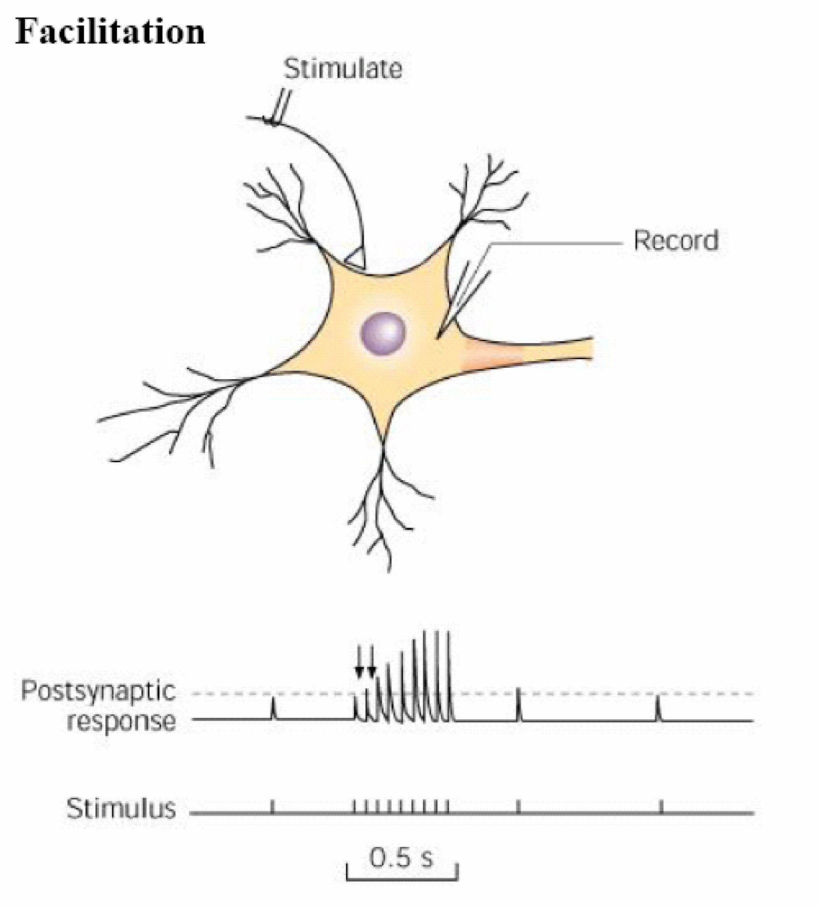
— e.g.,

Allen & Stevens, *PNAS* **1994**
Zador, *J. Neurophysiol.* **1998**
Abbott & Regehr, *Nature* **2004**

Activity–dependent (pre)synaptic kinetics: *depression* and *facilitation*

"Synaptic facilitation"

- repetitive activation, i.e., if synapse activated several times in quick succession produces an increase in amplitude of postsynaptic responses
  - to some maximum (up to signal x 7) characteristic for the synapse and stimulation frequency
- only some neurons show facilitation
- **mechanism already described by Katz & Miledi in 1968**



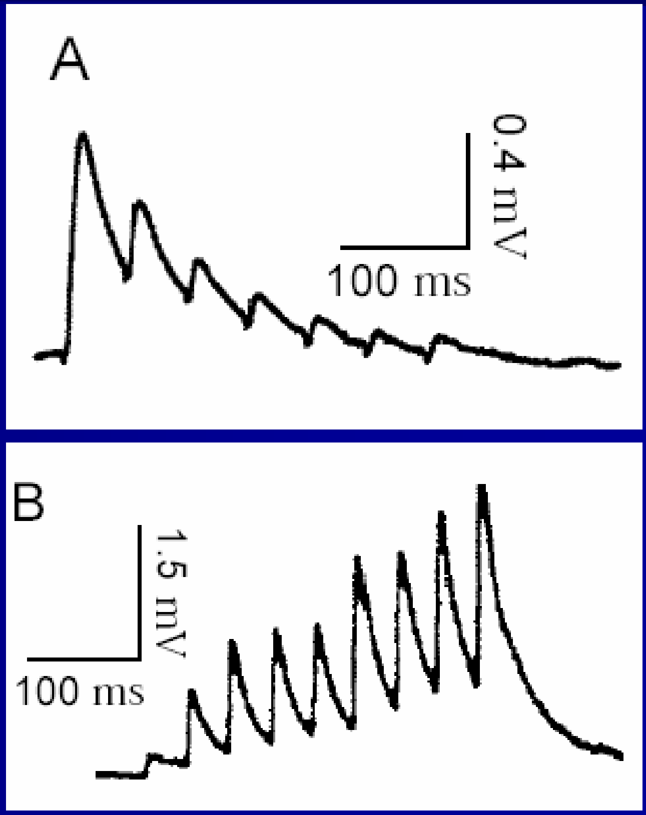
Activity–dependent (pre)synaptic kinetics: *depression* and *facilitation*

"Synaptic depression"

- In real circuits, periods of elevated activity may cause a noticeable decrease in size of postsynaptic response
- one mechanism for synaptic depression is **transmitter depletion**
- may compete with facilitation

Markram & Tsodyks, *PNAS* **1998**

Torres, Cortes, Marro & Kappen, *Neural Computat.* **2006, to appear**



Depression (A) and Facilitation (B)

Local fields $h_i(\boldsymbol{\sigma}, \mathbf{w}) = \sum_{j \neq i} w_{ij} \sigma_j$
depend on situation of interest; here:

- simplicity (let allow for some analytical result)
- closeness to Hopfield–Hebb neural net:

$$w_{ij} = N^{-1} \sum_{\mu} \xi_i^{\mu} \xi_j^{\mu}$$

pattern of activity

- let us try: $w_{ij} = \left[1 - (1 + \Phi)\, q(\boldsymbol{\pi})\right] N^{-1} \sum_{\mu=1}^{M} \xi_i^{\mu} \xi_j^{\mu}$

$$q(\boldsymbol{\pi}) \equiv (1 + \alpha) \sum_{\mu} \pi^{\mu}(\boldsymbol{\sigma})^2 \qquad \pi^{\mu}(\boldsymbol{\sigma}) = N^{-1} \sum_i \xi_i^{\mu} \sigma_i$$

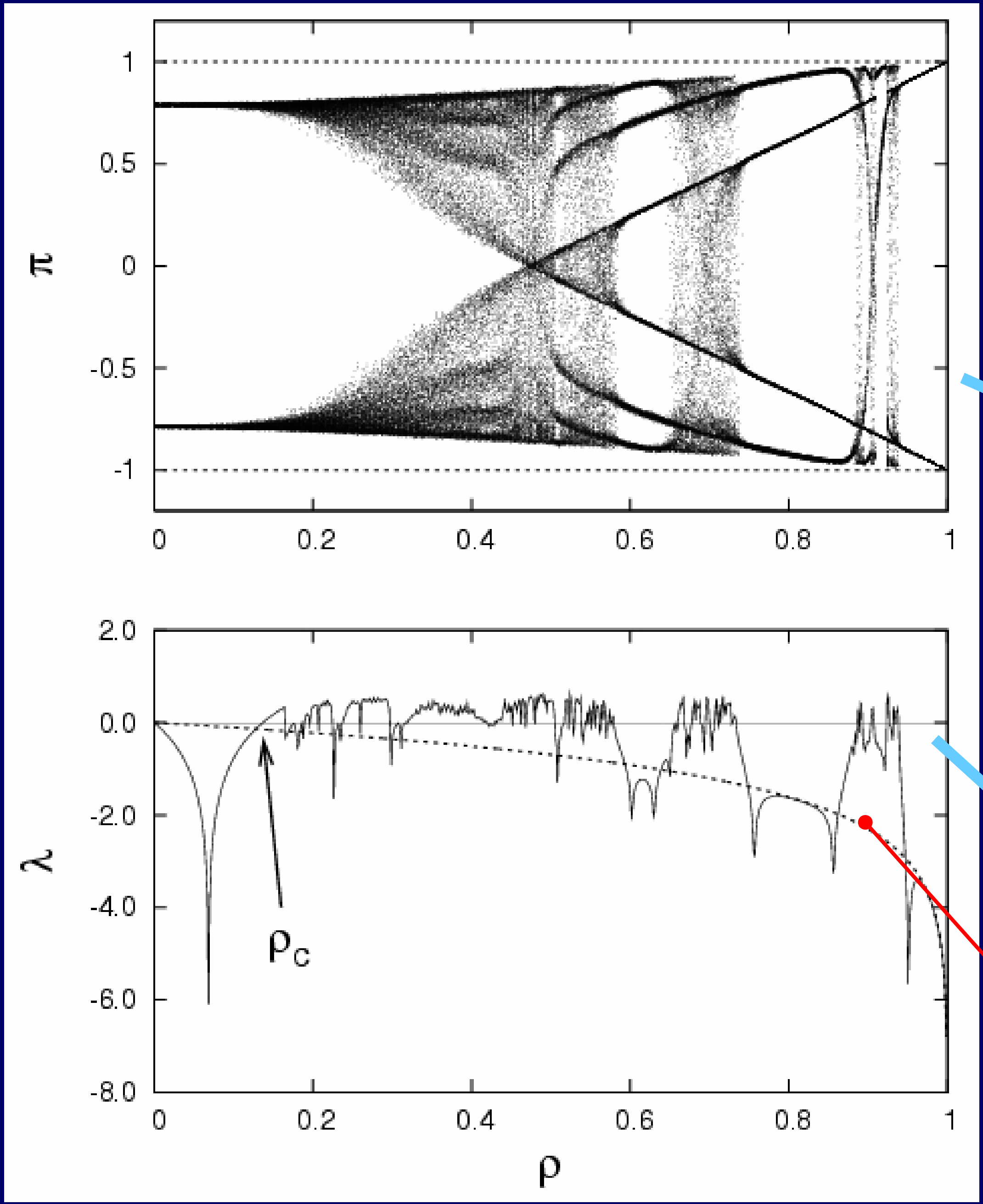
synapses are depressed by $-\Phi$ on average

- $\Phi = -1$ → Hopfield case (~ 4000 cites in SCI)

Main results:

- As in the Hopfield net, one may *store* patterns by (e.g.) Hebb's learning rule, and
  — these patterns are then attractors of dynamics
- However, the synaptic "noise" here importantly
  — destabilizes the attractors and, consequently:
  — susceptibility to external stimulus increases
  — an efficient search in attractors' space is induced

  this in close similarity with recent experim. observations
- Search may be chaotic for $\rho \geq \rho_C$, where even slight variations of ρ may switch search from regular to irregular



Evidence of destabilization of attractors, and transitions from regular to chaotic, as ρ is varied.

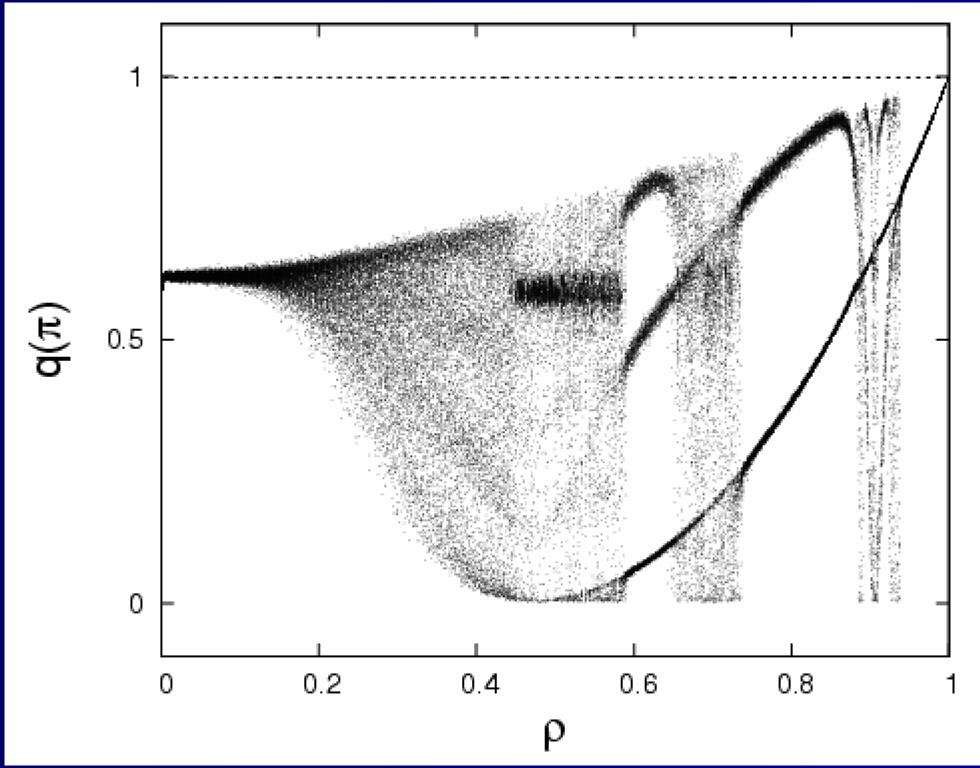
This shows the overlap between the current activity and a unique (randomly generated) pattern in a MC simulation for $\Phi = 0.5$, $N = 3600$ nodes and $\beta = 20$.

Lyapunov exponent for the same case, eventually becoming positive.

dashed line is Hopfield–Hebb result, i.e., $\Phi = -1$.

Same sort of behavior for *large* *M*

(M = 20 random patterns, N = 3600 nodes, Φ = 0.5, β = 20)

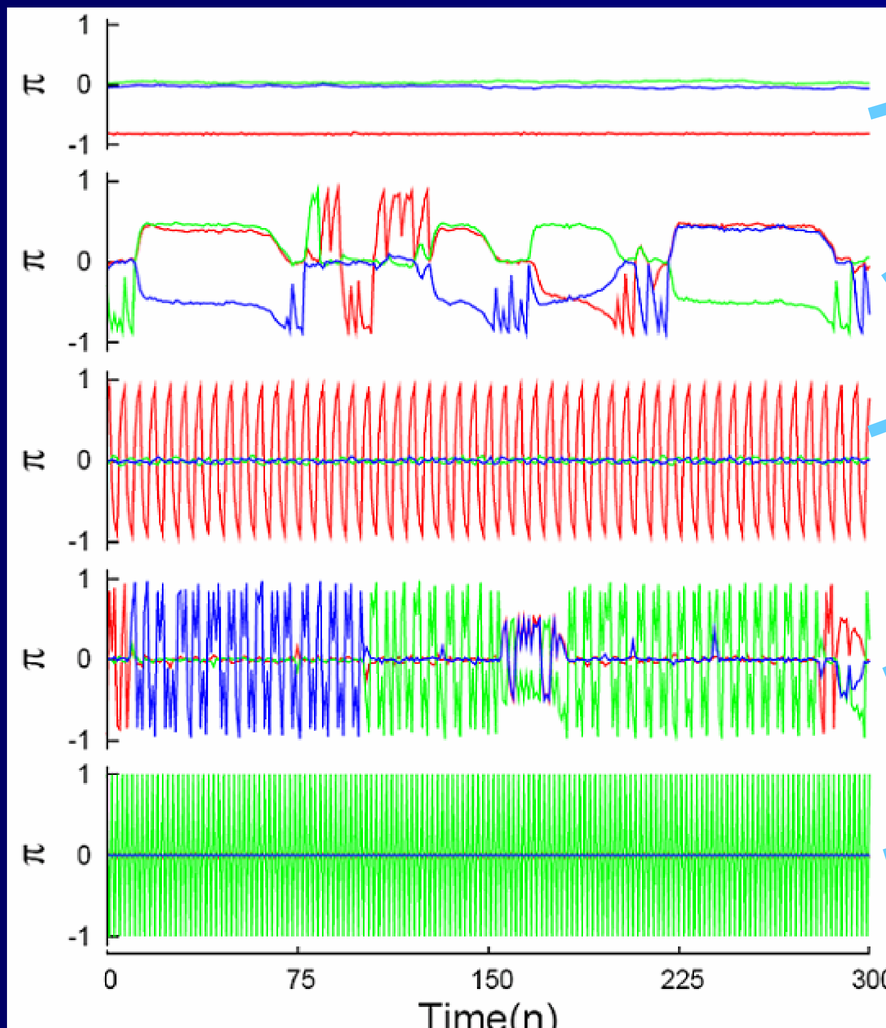


$$q(\pi) \equiv (1+\alpha) \sum_{\mu} \pi^{\mu} (\sigma)^2$$

Typical stationary (MC) runs

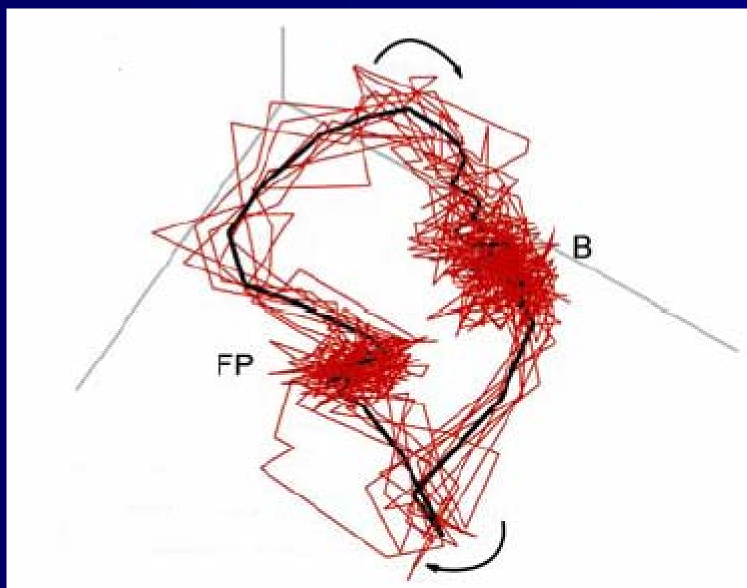
overlap versus time

(N = 1600, M = 3 uncorrelated patterns, Φ = 0.4, β = 20)



- **After convergence, stability of one of the attractors —in fact, an *anti-pattern* (zero overlap with the others)— for ρ = 0.08 < ρ_c = 0.085**
- **Fully irregular (positive Lyapunov exponent) behavior for ρ = 0.50 > ρ_c**
- **Regular oscillation between one attractor and its negative or anti-pattern for ρ = 0.65 > ρ_c**
- **Onset of chaos (again) as ρ is incresed somewhat; ρ = 0.92 in this case**
- **Rapid and ordered periodic oscillations between one pattern and its antipattern (all nodes synchronized)**

Experimobservationservati Expectation:



Response to odor stimuli of certain neurons in the locust antennal lobe, according to Mazor & Laurent, *Neuron* 48 2005

State of attention or, more specifically, “animals brain is exploring a sequence of states generating a specific pattern of activity that represents one specific odor”

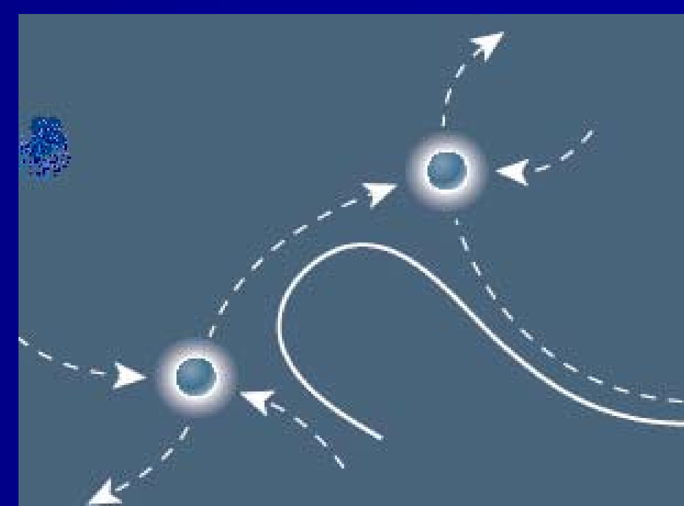
“instability inherent to chaotic motions facilitates system ability to move to any pattern at any time”

Hansel & Sompolinsky, *J. Comput. Neurosci.* **3** 1996

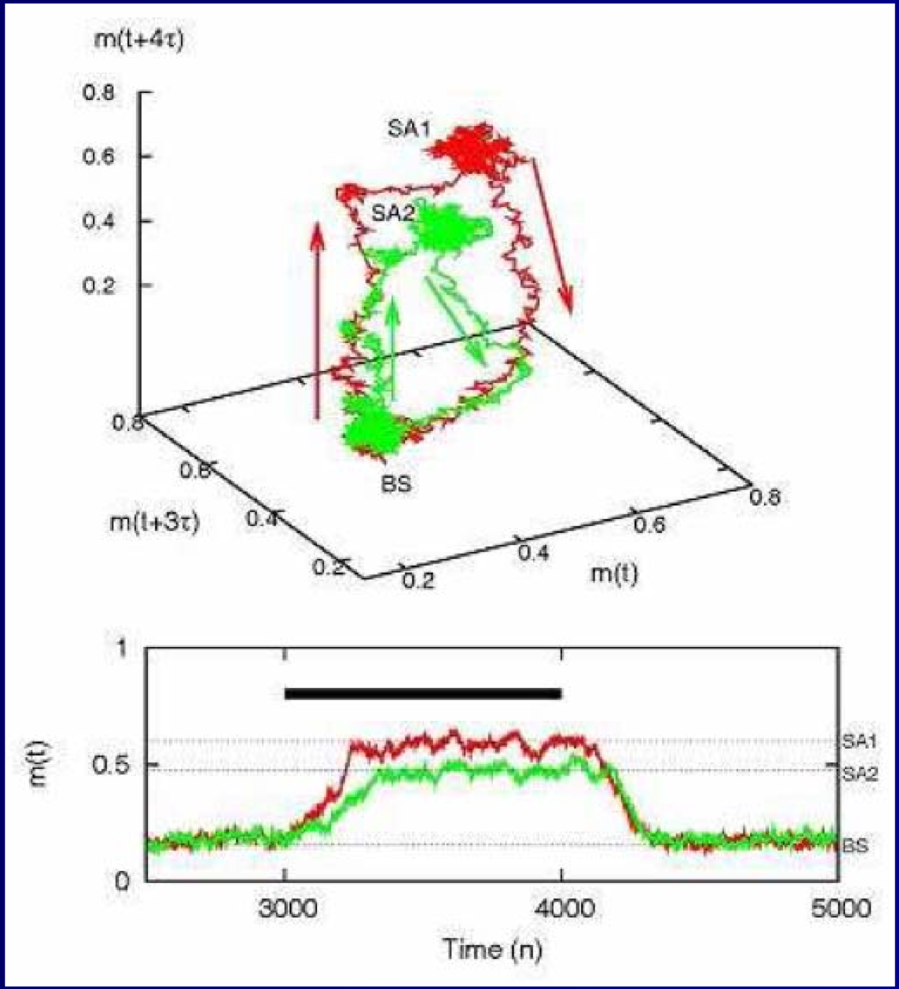
Korn & Faure, C. R. *Biologies* **326** 2003

Glass, in "Handbook of Brain Theory and Neural Networks", M.A. Arbib Ed., MIT Press 2003.

Ashwin & Timme, *Nature* **436** 2005

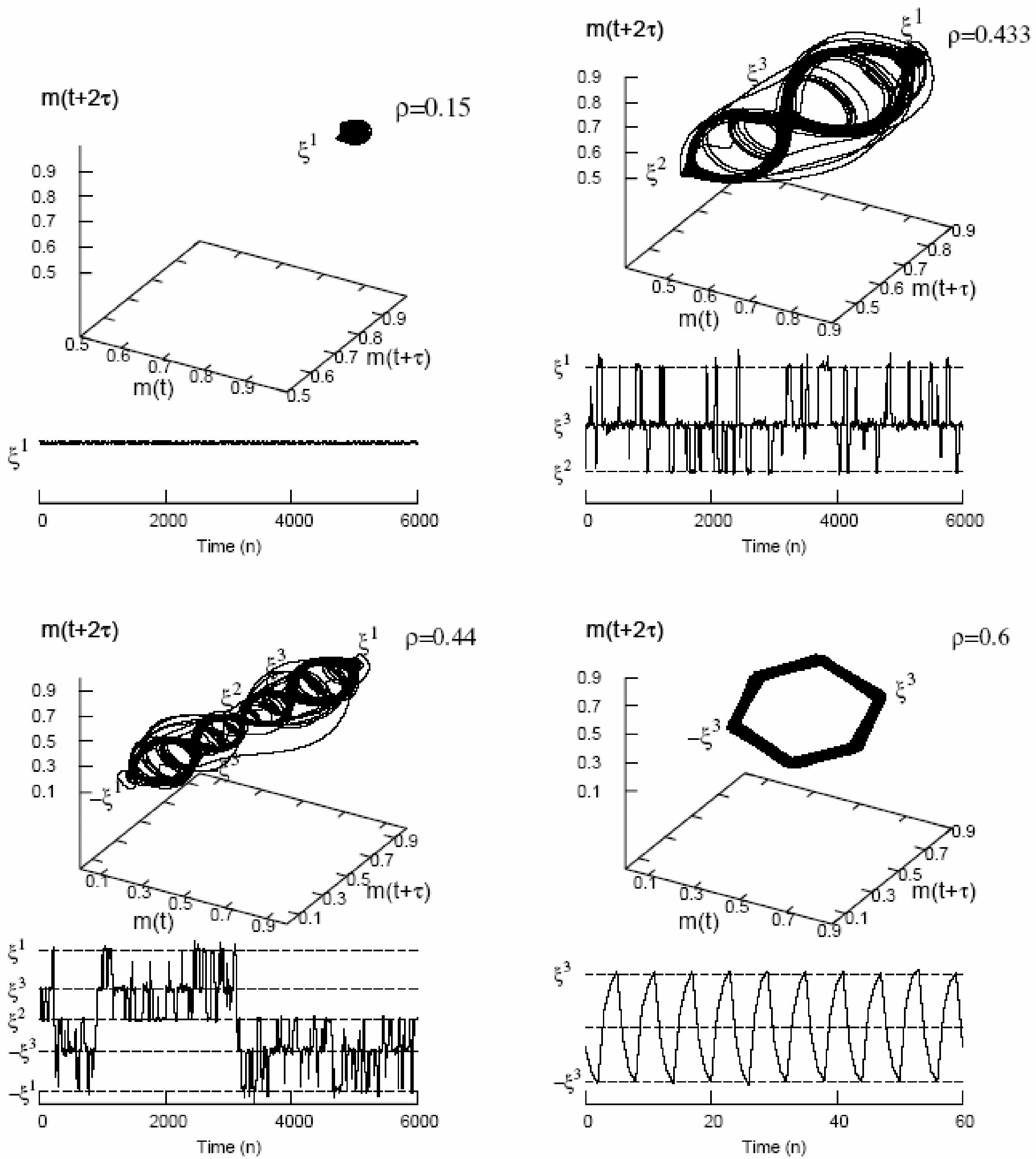


Model itinerancy induced by external stimuli



- **Phase space trajectories, and**
- ***t* variation of *mean firing rate*:**
  $m = (2N)^{-1} \sum_i (1+\sigma_i)$
- **This, trying to recreate <u>Mazor & Laurent observation</u>, shows two MC simulations for N = 1600, β = 4, Φ = 0.45, ρ = 3/64 < ρ_C and six patterns.**
- **Different stimuli of same intensity and duration correspond to green and red.**
- **There is stimulus destabilization in the absence of chaos.**

(Top graph involves standard false-neighbor method with embedding $d = 5$ and time delay $\tau = 20$.)



Chaotic switching among attractors

— simulates states of attention in the brain, and illustrates possible role of chaos

number of attractors visited increases with *ρ*

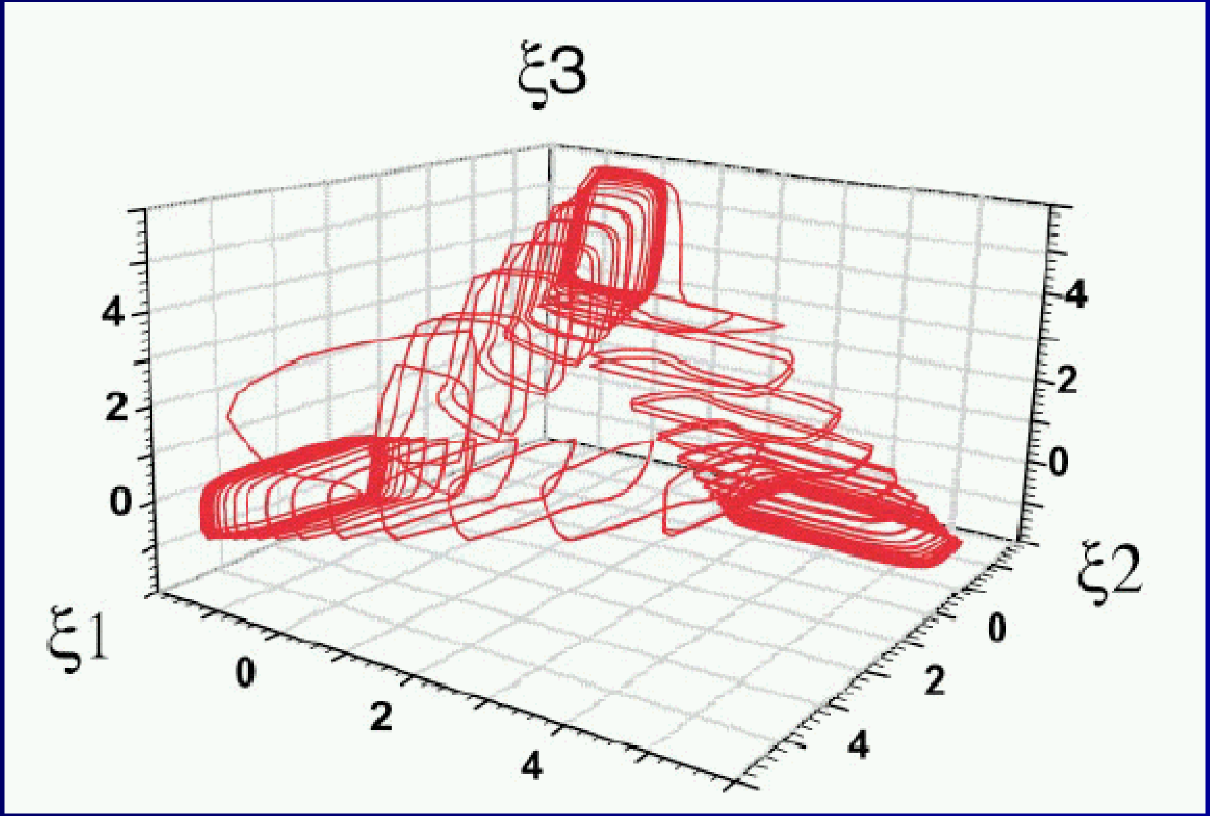
— until activity settles down to a periodic jumping between one of the patterns and its anti-pattern.

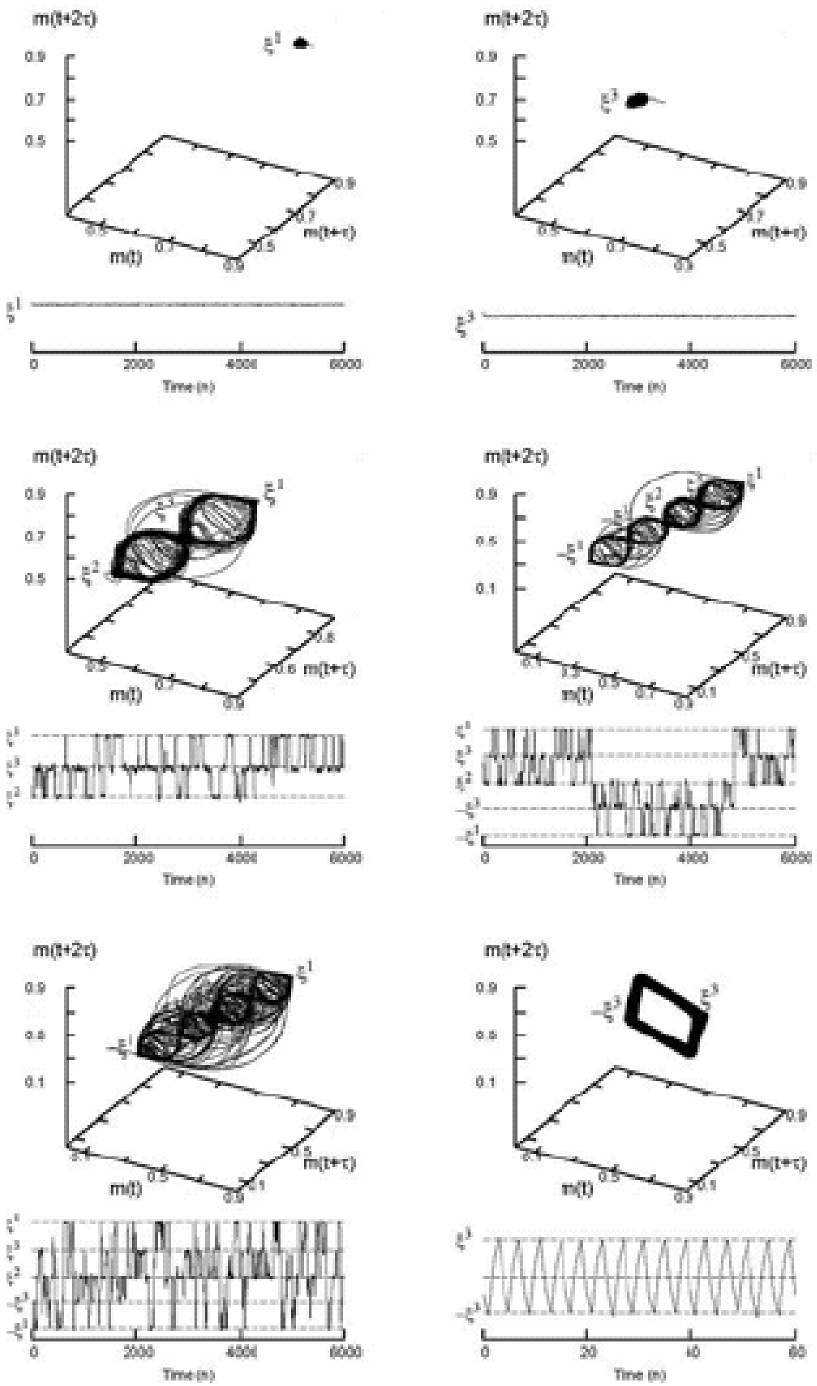
mean firing rate

$$m = \frac{1}{2N}\sum_i (1+\sigma_i)$$

versus *t* n and phase space trajectories

$\Phi = ½$, $N = 1600$, $\beta = 167$, $\rho_C = 0.38$, and three patterns, ξ^{μ} $\mu = 1,2,3$.


ξ3
4
2
0
ξ1
0
2
4
4
2
0
ξ2
0
2
4



Chaotic itinerancy

- same for other patterns and

  $\rho \;=\; 0.1,\; 0.3 \;<\; \rho_C = 0.38$

  wandering around one pattern; choice depends on initial condition

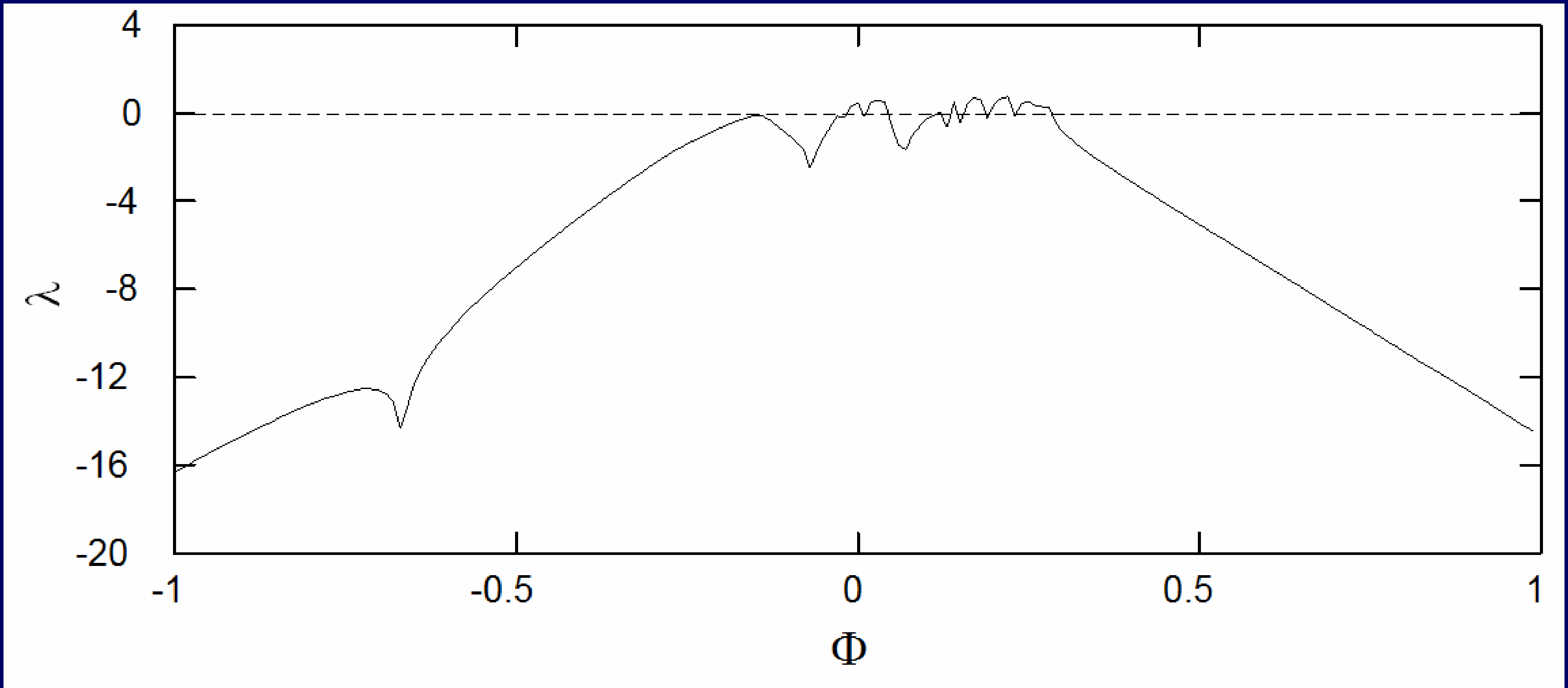
  $\rho \;=\; 0.384,\; 0.39,\; 0.4 \;>\; \rho_C$;

- visits all patterns and, for ρ large enough even the anti-patterns; as ρ increases, activity jumps to more distant, less correlated patterns

- finally, surpasses equi-probability of patterns, abandons chaotic regime to fall into a limit cycle

(phase space trajectories in graphs, using "standard false-neighbours" technique with $5n$ time delay and embedding dimension of 5)

Parallel updating, $\rho = 1$



Lyapunov exponent

for $M = 1$, $N = 10^5$ and $T = 0.1$

⇨ **complex (ρ,Φ) space!**

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