

Dynamics of Interacting Two Particle Species on Scale-Free Networks

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Outline

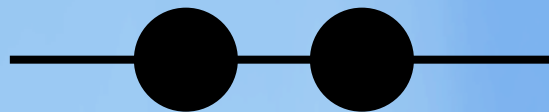
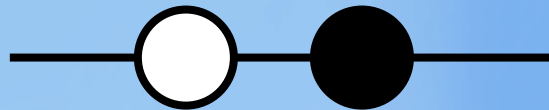
- Model for epidemic spreading
- Epidemic spreading on Scale-Free Networks
- Two species epidemic spreading on SF networks

References :

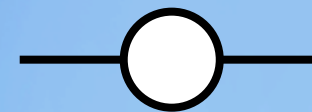
- 1 . "Asymmetrically Coupled Directed Percolation Systems" by JD Noh and H. park, Phys. Rev. Lett. **94**, 145702 (2005).
2. "Epidemic Dynamics of Interacting Two Particle Species on Scale-Free Networks", by Y.-Y. Ahn, N. Masuda, H. Jeong, and JD Noh, cond-mat/0608461 (2006).

Contact Process or Susceptible-Infected-Susceptible model

- Epidemic dynamics

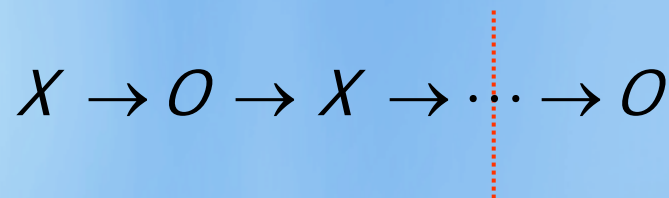


infection
($OX \rightarrow XX$)

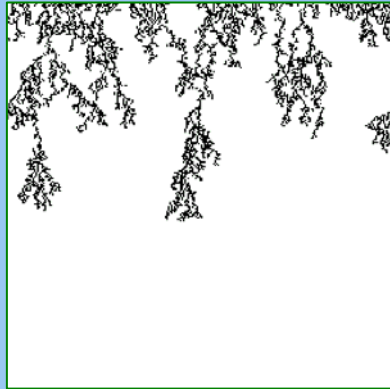
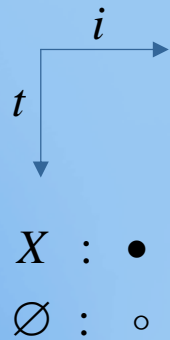


spontaneous healing
($X \rightarrow O$)

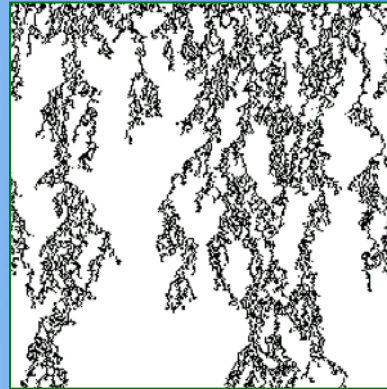
- Individual



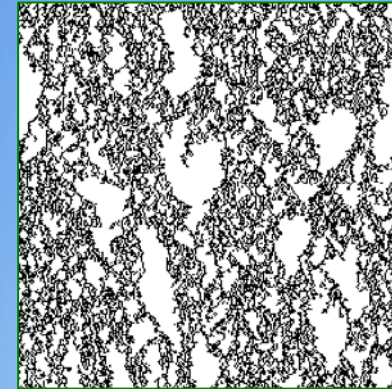
Non-Eq. Phase Transition



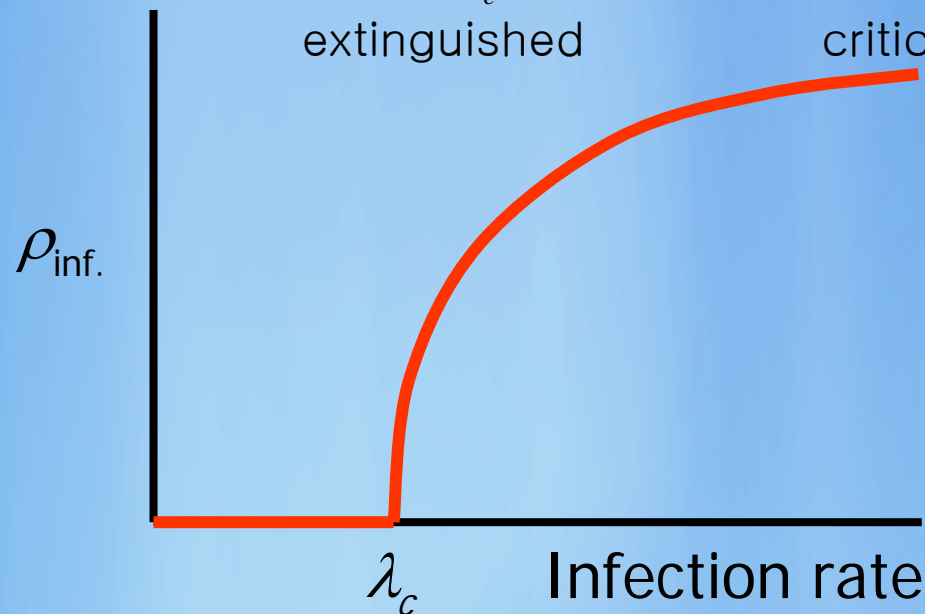
$\lambda < \lambda_c$
 extinguished



$\lambda = \lambda_c$
 critical



$\lambda > \lambda_c$
 everlasting



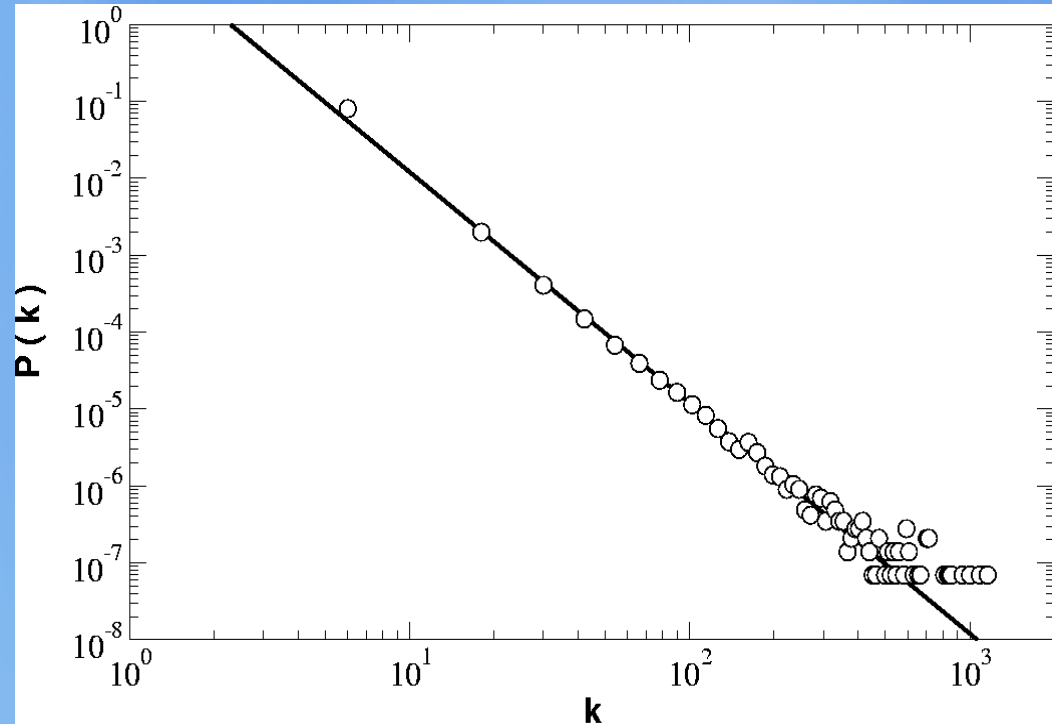
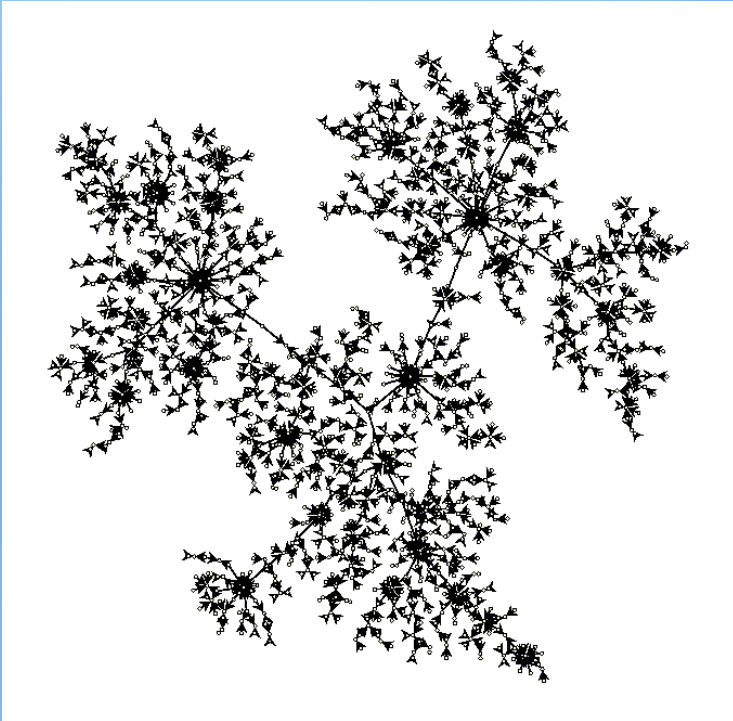
$$\rho_{\text{inf.}} \sim (\lambda - \lambda_c)^\beta$$

$$\xi_x \sim |\lambda - \lambda_c|^{-\nu_x}$$

$$\xi_t \sim |\lambda - \lambda_c|^{-\nu_t}$$

Directed Percolation
 universality class

Scale-Free Networks



Power-law degree distribution $P_{\text{deg.}}(k) \sim k^{-\gamma}$

Ex) Internet, WWW,...

Epidemic on SF Net. (single species)

- Dynamics
 - Spontaneous healing with rate 1
 - Infecting all neighboring nodes with rate λ
- E.g., computer viruses on the Internet
- Effect of the heterogeneity of the underlying networks?

Mean Field Theory

- ρ_k : infected node density among nodes with degree k

- Rate equation

$$\dot{\rho}_k = -\rho_k + \lambda k (1 - \rho_k) \Theta$$

$$\text{with } \Theta = \int \frac{dk'}{\langle k \rangle} k' \rho_{k'} P_{\text{deg.}}(k')$$

- Stationary condition

$$\dot{\rho}_k = 0 \Rightarrow \rho_k = \frac{\lambda k \Theta}{1 + \lambda k \Theta}$$

- Self-consistency equation

$$\Theta = \int \frac{dk}{\langle k \rangle} P_{\text{deg.}}(k) \frac{\lambda k^2 \Theta}{1 + \lambda k \Theta}$$

Mean Field Results

- From the SC equation $\Theta = \int \frac{dk}{\langle k \rangle} P_{\text{deg.}}(k) \frac{\lambda k^2 \Theta}{1 + \lambda k \Theta}$
- Epidemic threshold $\lambda_c = \langle k \rangle / \langle k^2 \rangle$
- For SF networks with $P_{\text{deg.}}(k) \sim k^{-\gamma}$
 - $\gamma > 3$: finite $\langle k^2 \rangle \rightarrow$ finite threshold
 - $\gamma \leq 3$: infinite $\langle k^2 \rangle \rightarrow \lambda_c = 0$

Immunization strategy?

Two species epidemics

- Epidemic spreading = competing process between virus (antigen) and vaccine (antibody)
- Competing asymmetric interaction
 - “A” induces “B”
 - “B” suppresses “A”
- Phase transitions and critical phenomena

[J.D. Noh and H. park, Phys. Rev. Lett. **94**, 145702 (2005).]

Two species epidemics on SF Networks.

- Dynamics
 - A particles
 - Spontaneous annihilation ($A \rightarrow 0$)
 - Infection ($OA \rightarrow AA$)
 - B particles
 - Spontaneous annihilation ($B \rightarrow 0$)
 - spreading ($OB \rightarrow BB$)
 - Interaction
 - Activation ($A \rightarrow AB$)
 - Healing ($AB \rightarrow B$)

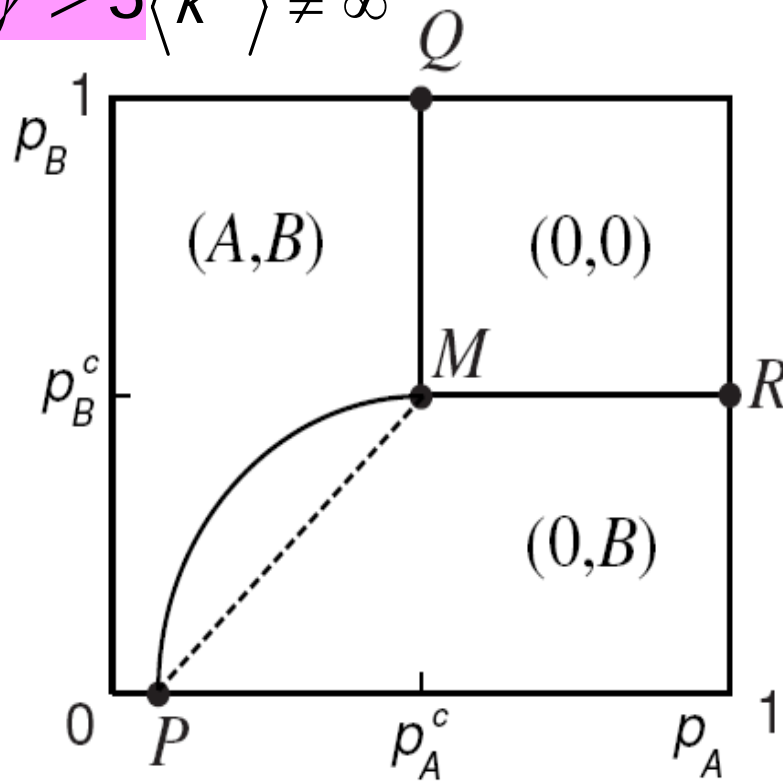
Mean Field Theory

- a_k : A particle density on nodes with degree k
 b_k : B particle density on nodes with degree k
- Rate equation

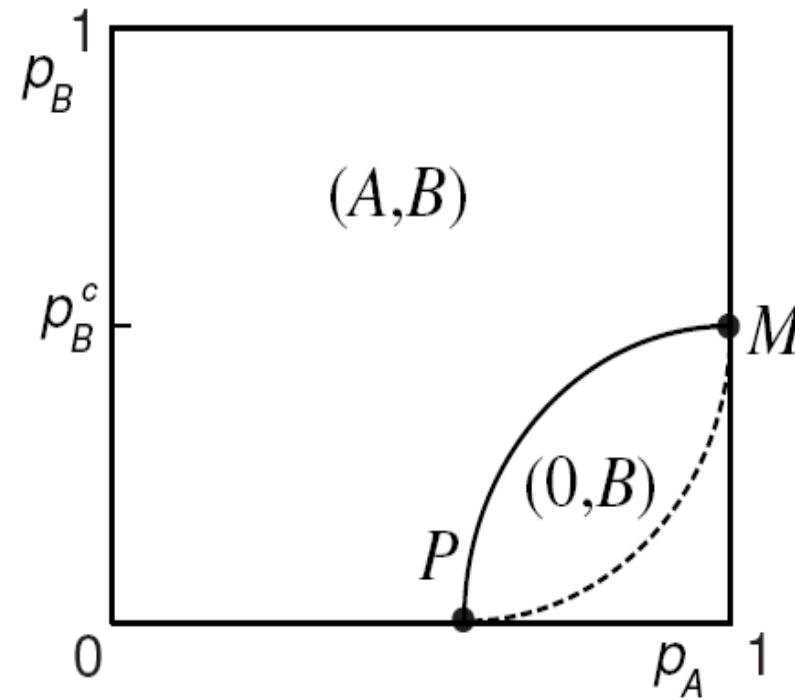
$$\begin{aligned}\dot{a}_k &= -p_A a_k + q_A(1 - \lambda)k(1 - a_k)\Theta_A(k) - \mu k a_k \Theta_B(k) \\ \dot{b}_k &= -p_B b_k + q_B k(1 - b_k)\Theta_B(k) + \lambda k(1 - b_k)\Theta_A(k),\end{aligned}$$

Phase Diagram

$\gamma > 3 \langle k^2 \rangle \neq \infty$



$\gamma \leq 3 \langle k^2 \rangle = \infty$



Summary

- Two species epidemic spreading model
- Critical behaviors
- Multi-species generalizations