Reflection, absorption and transmission of TE electromagnetic waves propagation in a nonuniform plasma slab

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Abstract

A numerical scheme is described to compute the reflection, absorption, and transmission coefficients of $\mathrm{TE}_{m,0}$ and plane electromagnetic waves propagating in a nonuniform plasma slab. The numerical solutions are in good agreement with theoretical solutions for two different kinds of electron density profiles. The solutions for various electron density profiles will be shown in $\mathrm{TE}_{m,0}$ mode. The method has applications to predicting the fields in plasmas where the density is profile is known, such as in microwave breakdown in a waveguide.

Key words: Electromagnetic waves; Reflection, absorption, and transmission; Plasma; Nonuniform

1. Introduction

As EM waves propagate from one homogeneous medium to another, they experience a change of the wave impedance at the interface. The impedance mismatch generally leads to the reflection, absorption, and transmission of EM waves. Unlike common optical media, nonuniform plasma is an inhomogeneous and lossy medium. The general expression for the reflection, absorption, and transmission of EM waves in a plasma with arbitrary density profile cannot be expressed in a closed form. Moreover, since the plasma consists of free charged particles, its state is significantly influenced by EM waves, which leads to further effects on EM waves. Thus, to describe the interaction of charged particles and EM waves self-consistently, the spatial and temporal information of the fields inside a plasma is indispensable.

In this paper, a numerical scheme is described to compute the reflection, absorption, and transmission of plane and $\mathrm{TE}_{m,0}$ EM waves propagating in a plasma slab inside a waveguide. The method will be obtained for a plasma slab with arbitrary density profile of the finite thickness in the propagation (longitudinal) direction, including a nonconducting neutral gas background. The variation of the profile along the transverse direction will be neglected compared to the longitudinal variation. Collisions between charged particles and neutral gas can include elastic scattering, excitation, ionization, and charge exchange. It will be assumed that the magnetic forces on charged particles are negligible compared with the electric forces; that is, the plasma is in the classical regime with no external magnetic fields. The waveguide will be assumed to be perfectly conducting.

2. Wave Equation

A wave propagating with a time dependence of e^{-jwt} satisfies the following wave equation:

$$(\nabla^2 + k^2)\mathbf{B} = \mathbf{0} \ , \tag{1}$$

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where k is the wave number, $k^2 = \mu\epsilon w^2$. μ and ϵ are the permeability and permittivity, respectively. When k is uniform along the transverse direction, the method of separation of variables can be applied. Applying boundary conditions on conducting surfaces for the rectangular waveguide with the cross section of L_x and L_y, B_z can be given in the following form:

$$B_z = A\cos(k_{x,m}x)\cos(k_{y,n}y)Z(z) \ , \tag{2}$$

where $k_{x,m} = m\pi/L_x$ and $k_{y,n} = n\pi/L_y$. m and n are the number of half-waves in the wider and narrower dimensions, respectively. A is the amplitude of the incident wave. A new function $Z(z)$ should satisfy the following equation:

$$\frac{\partial^2 Z}{\partial z^2} + k_z^2 Z = 0 \ , \tag{3}$$

where $k_z^2 = k^2 - \gamma_{m,n}^2$ and $\gamma_{m,n}^2 = k_{x,m}^2 + k_{y,n}^2$.

Consider a waveguide filled with 3 media along the direction of propagation: medium 1, medium 2, and medium 3. In semi-infinite homogeneous lossless media 1 and 3, k is constant with k_1 and k_3, respectively. Medium 2 with $k_2(z)$ is the plasma extending from $z = z_1$ to $z = z_2$. As a generalization of [1] to the case with multiple species, the wave number in medium 2 is complex as follows:

$$k_2\,(z) = k_{gas}\left(1 - \sum_i \frac{w_{p,i}^2(z)}{w(w + j\nu_{m,i}(z))}\right)^{1/2} \ , \tag{4}$$

where i denotes the index of charged species. $w_{p,i} = (n_i q_i^2/\varepsilon_0 m_i)^{1/2}$ is the plasma frequency and $\nu_{m,i}$ is the momentum transfer frequency with the neutral gas. The density n_i and the collision frequency $\nu_{m,i}$ have the dependence on z coordinate. k_{gas} is the wave number in the nonconducting gas in medium 2.

In medium 1, $Z_1(z)$ satisfying Eq. (3) consists of incident and reflected waves as follows:

$$Z_1(z) = e^{jk_{z,1}z} - \Gamma e^{-jk_{z,1}z} \ , \tag{5}$$

where Γ is the reflection coefficient to be determined from the boundary condition at the interface between media 1 and 2. $Z_3(z)$ in medium 3 has the following form:

$$Z_3(z) = \tau e^{jk_{z,3}(z-z_2)} \ , \tag{6}$$

where τ is the transmission coefficient to be determined from the boundary condition at the interface between media 2 and 3.

In TE$_{m,0}$ mode, $\gamma_{m,0}^2 = k_{x,m}^2$ and E_y is given as follows:

$$E_y(z) = A\frac{jw}{k_{x,m}}Z(z)\sin(k_{x,m}x) \ . \tag{7}$$

If the amplitude of the electric field of the incident wave is given as E_0, $A = E_0 k_{x,m}/jw$. Applying the boundary conditions at the interface between media 1 and 2 with an assumption of $\mu_1 = \mu_2$, we obtain:

$$e^{jk_{z,1}z_1} - \Gamma e^{-jk_{z,1}z_1} = Z(z = z_1^+) \tag{8}$$

and

$$jk_{z,1}(e^{jk_{z,1}z_1} + \Gamma e^{-jk_{z,1}z_1}) = \frac{\partial Z}{\partial z}(z = z_1^+) \ . \tag{9}$$

Combining two equations above, the following equations can be obtained:

$$2jk_{z,1}e^{jk_{z,1}z_1} = jk_{z,1}Z(z = z_1^+) + \frac{\partial Z}{\partial z}(z = z_1^+) \tag{10}$$

and

$$\Gamma = e^{jk_{z,1}z_1}\{e^{jk_{z,1}z_1} - Z(z = z_1^+)\} \ . \tag{11}$$

In a same way, applying the boundary conditions at the interface between media 2 and 3 with an assumption of $\mu_2 = \mu_3$, we obtain:

$$\tau = Z(z = z_2^-) \tag{12}$$

and

$$jk_{z,3}\tau = \frac{\partial Z}{\partial z}(z = z_2^-) \ . \tag{13}$$

Combining the two equations above, the following equation can be obtained:

$$jk_{z,3}Z(z = z_2^-) = \frac{\partial Z}{\partial z}(z = z_2^-) \ . \tag{14}$$

In medium 2, Eq. (3) is

$$\frac{\partial^2 Z_2}{\partial z^2} + k_{z,2}^2 Z_2 = 0 \ . \tag{15}$$

Equation (7) can be expressed in the following form:

$$E_y(z) = E_0 Z(z)\sin(k_{x,m}x) \ . \tag{16}$$

This is an alternative definition of $Z(z)$. Although it has been derived for TE$_{m,0}$ EM waves, we find that Eqs. (10)–(12) and (14)–(15) also hold for plane waves with $E_y(z) = E_0 Z(z)$ by substituting $m = 0$ ($\gamma_{0,0} = 0$ and $k^2 = k_z^2$).

The reflectance, transmittance, and absorbance can be obtained from the reflection and transmission coefficients as follows: $R = |\Gamma|^2$, $T = k_{z,3}|\tau|^2/k_{z,1}$, and $A = 1 - R - T$.

3. Finite Difference Equation

Using a centered difference scheme, the finite difference form of Eq. (15) is

$$(Z_2)^{j-1} + \{-2 + (k_{z,2}^2)^j(\Delta z)^2\}(Z_2)^j + (Z_2)^{j+1} = 0\,, \qquad (17)$$

where $j = 1, 2, ..., nc-1$ and nc is the number of cells in the uniformly gridded space with a fixed cell size Δz. The numerical solution $(Z_2)^j$ is third-order accurate. Equations (10) and (14) have the derivative form at the interface. At $z = z_1 (=z^0)$, the following relationship can be obtained from Eq. 3:

$$\frac{\partial Z_2}{\partial z}(z = z_1^+ + 0.5\Delta z) - \frac{\partial Z_2}{\partial z}(z = z_1^+) = -k_{z,2}^2(z = z_1^+)\int_{z_1^+}^{z_1^+ + 0.5\Delta z} Z_2(z)dz \qquad (18)$$

It can be expressed in the following form:

$$\frac{\partial Z_2}{\partial z}(z = z_1^+) = \frac{(Z_2)^1 - (Z_2)^0}{\Delta z} + \frac{3(Z_2)^0 + (Z_2)^1}{8}k_{z,2}^2(z = z_1^+)\Delta z\,. \qquad (19)$$

In the same way, at $z = z_2 (=z^{nc})$,

$$\frac{\partial Z_2}{\partial z}(z = z_2^-) = \frac{(Z_2)^{nc} - (Z_2)^{nc-1}}{\Delta z} - \frac{3(Z_2)^{nc} + (Z_2)^{nc-1}}{8}k_{z,2}^2(z = z_2^-)\Delta z\,. \qquad (20)$$

Using above equations, Eqs. (10) and (14) can be expressed in the following form:

$$\{jk_{z,1}\Delta z - 1 + \frac{3}{8}k_{z,2}^2(z = z_1^+)(\Delta z)^2)\}(Z_2)^0 + \{1 + \frac{1}{8}k_{z,2}^2(z = z_1^+)(\Delta z)^2\}(Z_2)^1 = 2jk_{z,1}\Delta z e^{jk_{z,1}z_1} \qquad (21)$$

$$\{1 + \frac{1}{8}k_{z,2}^2(z = z_2^-)(\Delta z)^2)\}(Z_2)^{nc-1} + \{jk_{z,3}\Delta z - 1 + \frac{3}{8}k_{z,2}^2(z = z_2^-)(\Delta z)^2\}(Z_2)^{nc} = 0 \qquad (22)$$

The finite difference equations (17), (21), and (22) can be written in a tridiagonal matrix and solved numerically.

4. Results

In Fig. 1, the normalized wave number $k_{z,2}/k_{gas}$ in medium 2 in Eq. (4) is shown as a function of

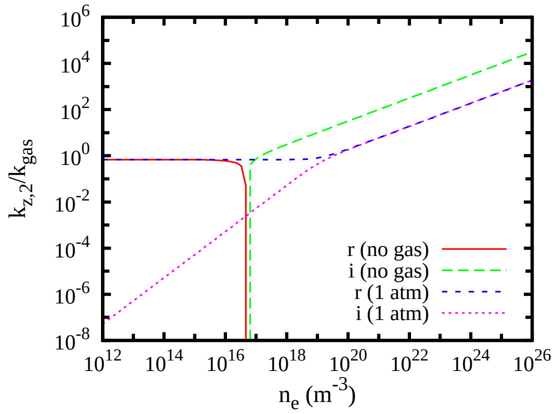

Fig. 1. Real (r) and imaginary (i) parts of $k_{z,2}/k_{gas}$ as a function of the electron density in a rectangular waveguide without any background gas and with argon gas of 1 atm. $w/2\pi = 2.85$ GHz and $\gamma_{m,n}/k_{gas} = 0.73$.

the electron density. When there exist only electrons without any background gas, the EM wave is totally reflected at $z = z_1$ if the wave frequency (w) is equal to or less than the cutoff frequency $w_{p,e}$. On the other hand, when there is a background gas colliding with electrons, the wave attenuates in medium 2.

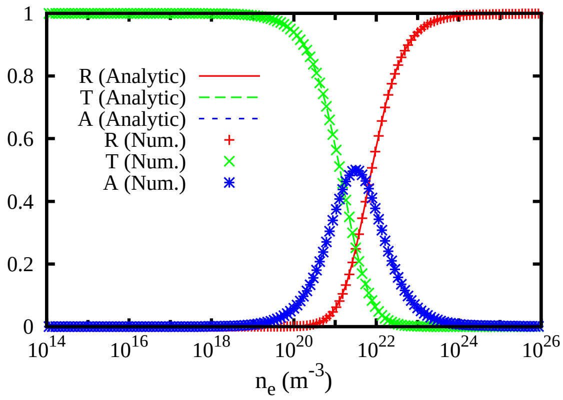

Fig. 2. Reflectance (R), transmittance (T), and absorbance (A) as a function of the uniform electron density under argon gas of 1 atm in a rectangular waveguide. $w/2\pi = 2.85$ GHz, $z_2 - z_1 = 100\ \mu$m, $k_1 = k_{gas} = k_3 = w/c$, and $\gamma_{m,0}^2 = 1.9 \cdot 10^3$ m^{-2}.

To validate the numerical solution of the wave equation, results have been compared with theoretical results under two different kinds of electron density profiles. The first is the case of a TE$_{m,0}$ wave propagating in a uniform density electron plasma. The reflectance, transmittance, and absorbance of TE$_{m,0}$ mode have been analytically derived. As shown in Fig. 2, the numerical solution compares well with analytic solution. The second is the case of a plane wave incident on a linear ramp of elec-

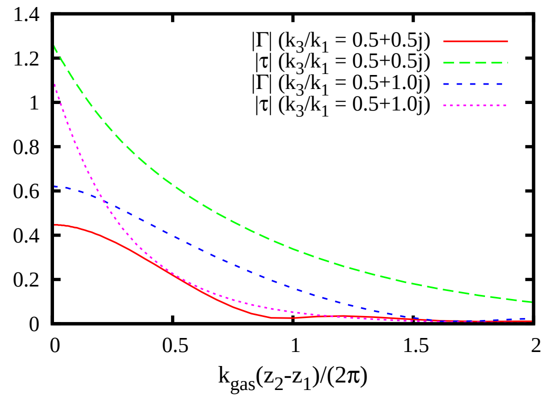

Fig. 3. Magnitudes of reflection and transmission coefficients for a plane wave as a function of the ramp width $z_2 - z_1$. $k_1 = k_{2,gas} = k_{3,gas} = w/c$.

tron density. In this case, medium 3 is filled with uniform density electrons with a background gas. The magnitudes of the reflected and transmitted coefficients were obtained as a function of the ramp width for different values of k_3/k_1. Our result in Fig. 3 is in good agreement with that of Ref. [2].

In Fig. 4, the reflectance, transmittance, and electric field profile are shown for a $\mathrm{TE}_{m,0}$ wave for electrons with the density profile of uniform, triangular, parabolic, and exponential shapes, keeping the spatially averaged electron density constant. The results clearly depend on the spatial profile of the plasma.

5. Summary

The reflection, absorption, and transmission of $\mathrm{TE}_{m,0}$ and plane EM waves propagating in a nonuniform plasma slab have been analyzed using a numerical scheme. Our numerical solutions show good agreement with theoretical solutions for two different kinds of electron density profiles: uniform density in $\mathrm{TE}_{m,0}$ mode and linear ramp density for a plane wave. The dependence of solutions on the electron density profile has been shown in $\mathrm{TE}_{m,0}$ mode.

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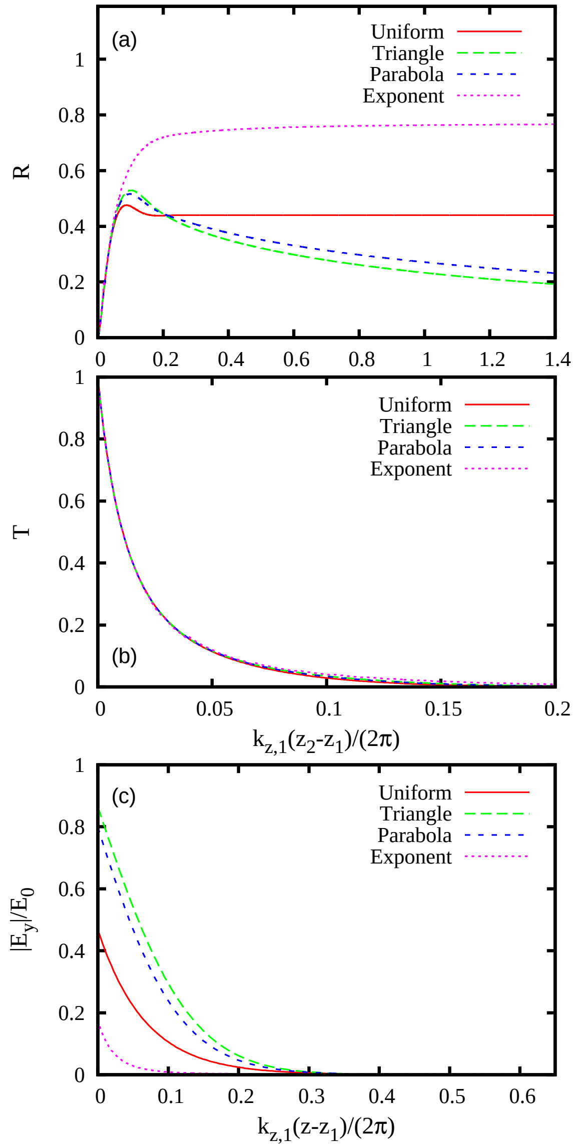

Fig. 4. (a) Reflectance, (b) transmittance, and (c) electric field distribution (at $k_{x,m}x = \pi/2$ and with $z_2 - z_1 = 0.1$ m) for $\mathrm{TE}_{m,0}$ wave under argon gas of 1 atm in a rectangular waveguide. $w/2\pi = 2.85$ GHz, $k_1 = k_{gas} = k_3 = w/c$, $\gamma^2_{m,0} = 1.9 \cdot 10^3$ m^{-2}, and $n_{e,ave} = 7.9 \cdot 10^{19}$ m^{-3}.

References

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